2

Elementary Network Theorems

2.1 INTRODUCTION

In Chapter 1, we have studied basic network concepts. In network analysis, we have to find currents and voltages in various parts of networks. In this chapter, we will study elementary network theorems like Kirchhoff's laws, mesh analysis and node analysis. These methods are applicable to all types of networks. The first step in analyzing networks is to apply Ohm's law and Kirchhoff's laws. The second step is the solving of these equations by mathematical tools.

2.2 KIRCHHOFF'S LAWS

The entire study of electric network analysis is based mainly on Kirchhoff's laws. But before discussing this, it is essential to familiarise ourselves with the following terms:

Node A node is a junction where two or more network elements are connected together.

Branch An element or number of elements connected between two nodes constitute a branch.

Loop A loop is any closed part of the circuit.

Mesh A mesh is the most elementary form of a loop and cannot be further divided into other loops. All meshes are loops but all loops are not meshes.

1. ***Kirchhoff's Current Law (KCL)*** The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

   Consider five conductors, carrying currents I_1, I_2, I_3, I_4 and I_5 meeting at a point O as shown in Fig. 2.1. Assuming the incoming currents to be positive and outgoing currents negative, we have

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

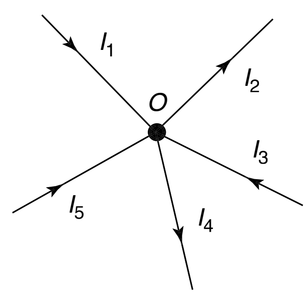

Fig. 2.1 *Kirchhoff's current law*

   Thus, the above law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

2. ***Kirchhoff's Voltage Law (KVL)*** The algebraic sum of all the voltages in any closed circuit or mesh or loop is zero.

   If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of emfs and the sum of voltage drops or rises meeting on the way is zero.

3. *Determination of Sign* A rise in potential can be assumed to be positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(i) If we go from the positive terminal of the battery or source to the negative terminal, there is a fall in potential and so the emf should be assigned a negative sign (Fig. 2.2a). If we go from the negative terminal of the battery or source to the positive terminal, there is a rise in potential and so the emf should be given a positive sign (Fig. 2.2b).

Fig. 2.2 *Sign convention*

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in the potential and so the sign of this voltage drop is negative (Fig. 2.3a). If we go opposite to the direction of the current flow, there is a rise in potential and hence, this voltage drop should be given a positive sign (Fig. 2.3b).

Fig. 2.3 *Sign convention*

Example 2.1 *In Fig. 2.4, the voltage drop across the 15 Ω resistor is 30 V, having the polarity indicated. Find the value of R.*

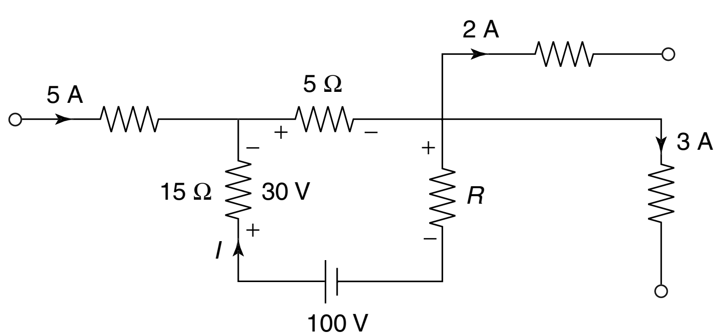

Fig. 2.4

Solution Current through the 15 Ω resistor

$$I = \frac{30}{15} = 2 \text{ A}$$

Current through the 5 Ω resistor = 5 + 2 = 7 A

Applying KVL to the closed path,

$$-5(7) - R(2) + 100 - 30 = 0$$
$$-35 - 2R + 100 - 30 = 0$$
$$R = 17.5\ \Omega$$

Example 2.2 *Determine the currents I_1, I_2 and I_3 in Fig. 2.5.*

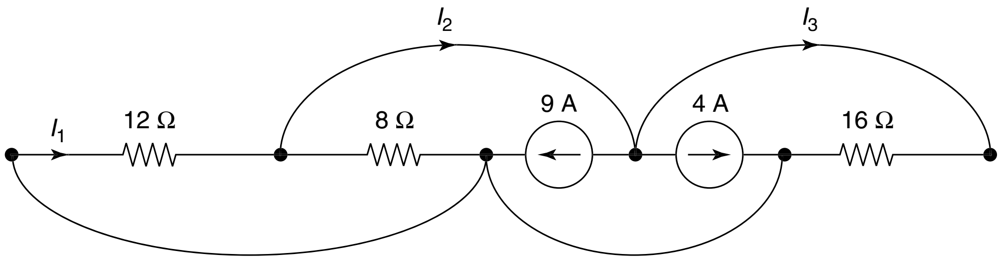

Fig. 2.5

Solution Assigning currents to all the branches (Fig. 2.6),

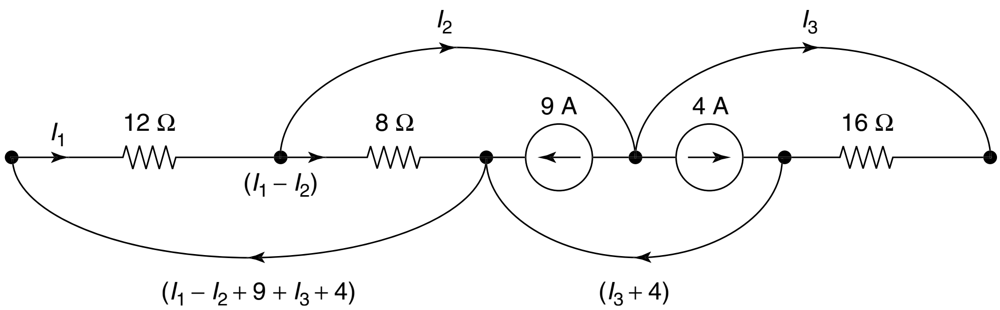

Fig. 2.6

From Fig. 2.6,

$$I_1 = I_1 - I_2 + 9 + I_3 + 4$$

$$I_2 - I_3 = 13 \qquad \text{...(i)}$$

Also,

$$-12 I_1 - 8(I_1 - I_2) = 0$$

$$-20 I_1 + 8 I_2 = 0 \qquad \text{...(ii)}$$

and

$$-12 I_1 - 16 I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 4 \text{ A}$$

$$I_2 = 10 \text{ A}$$

$$I_3 = -3 \text{ A}$$

Example 2.3 *Find currents in all the branches of the network shown in Fig. 2.7.*

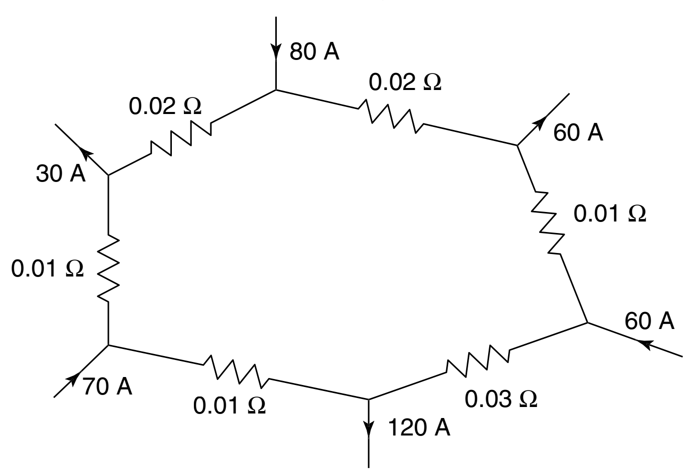

Fig. 2.7

Solution Let $I_{AF} = x$

Then $I_{FE} = x - 30$

$I_{ED} = x + 40$

$I_{DC} = x - 80$

$I_{CB} = x - 20$

$I_{BA} = x - 80$

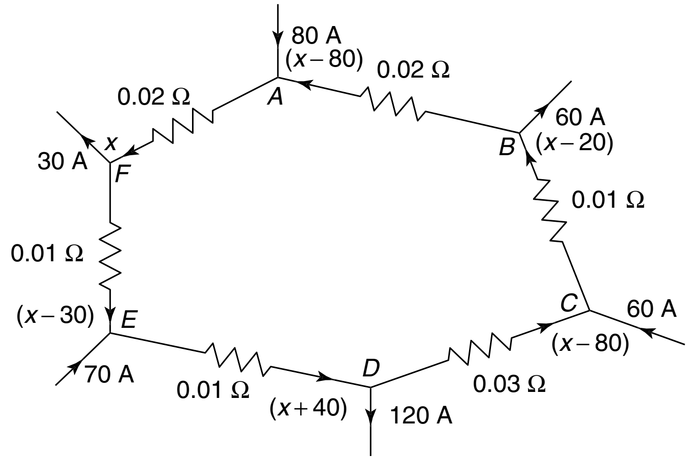

Fig. 2.8

Applying KVL to the closed path *AFEDCBA* (Fig. 2.8),

$$-0.02x - 0.01(x-30) - 0.01(x+40) - 0.03(x-80) - 0.01(x-20) - 0.02(x-80) = 0$$

$$x = 41 \text{ A}$$

$$I_{AF} = 41 \text{ A}$$

$$I_{FE} = 41 - 30 = 11 \text{ A}$$

$$I_{ED} = 41 + 40 = 81 \text{ A}$$

$$I_{DC} = 41 - 80 = -39 \text{ A}$$

$$I_{CD} = 39 \text{ A}$$

$$I_{CB} = 41 - 20 = 21 \text{ A}$$

$$I_{BA} = 41 - 80 = -39 \text{ A}$$

$$I_{AB} = 39 \text{ A}$$

Example 2.4 *Find currents in all the branches of the network shown in Fig. 2.9.*

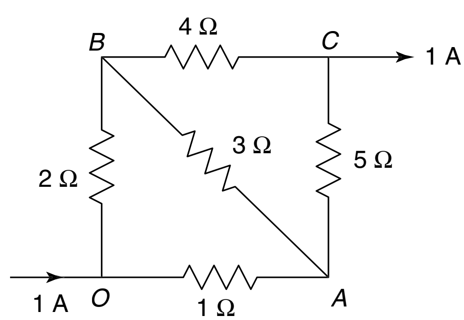

Fig. 2.9

Solution Assigning currents to all the branches (Fig. 2.10),
Applying KVL to the closed path *OBAO*,

$$-2(1-x) - 3y + 1(x) = 0$$

$$3x - 3y = 2 \qquad \ldots\text{(i)}$$

Applying KVL to the closed path *ABCA*,

$$3y - 4(1-x-y) + 5(x+y) = 0$$

$$9x + 12y = 4 \qquad \ldots\text{(ii)}$$

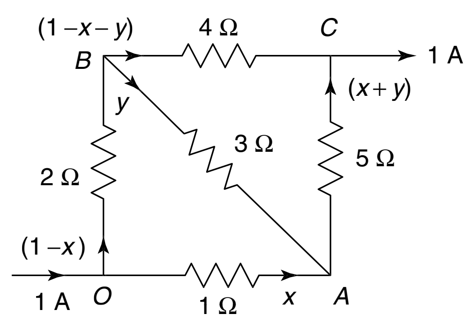

Fig. 2.10

Solving Eqs (i) and (ii),

$$x = 0.57 \text{ A}$$
$$y = -0.095 \text{ A}$$
$$I_{OA} = 0.57 \text{ A}$$
$$I_{OB} = 1 - 0.57 = 0.43 \text{ A}$$
$$I_{AB} = 0.095 \text{ A}$$
$$I_{AC} = 0.57 - 0.095 = 0.475 \text{ A}$$
$$I_{BC} = 1 - 0.57 + 0.095 = 0.525 \text{ A}$$

Example 2.5 *What is the potential difference between points x and y in the network shown in Fig. 2.11?*

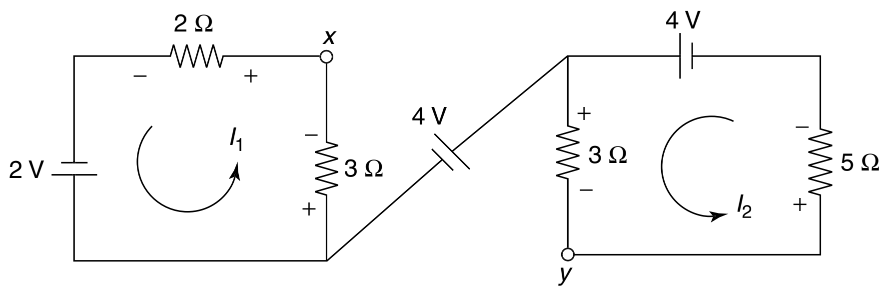

Fig. 2.11

Solution

$$I_1 = \frac{2}{2+3} = 0.4 \text{ A}$$
$$I_2 = \frac{4}{3+5} = 0.5 \text{ A}$$

Potential difference between points x and $y = V_{xy} = V_x - V_y$

Writing KVL equation for the path x to y,

$$V_x + 3I_1 + 4 - 3I_2 - V_y = 0$$
$$V_x + 3(0.4) + 4 - 3(0.5) - V_y = 0$$
$$V_x - V_y = -3.7$$
$$V_{xy} = -3.7 \text{ V}$$

Example 2.6 *Find the voltage between points A and B in Fig. 2.12.*

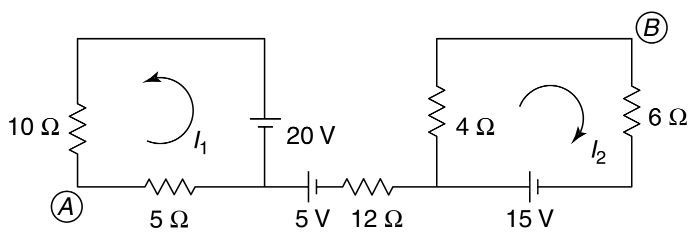

Fig. 2.12

Solution

$$I_1 = \frac{20}{10+5} = 1.33 \text{ A}$$

$$I_2 = \frac{15}{4+6} = 1.5 \text{ A}$$

From Fig. 2.13,
Voltage between points A and $B = V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 5I_1 - 5 - 15 + 6I_2 - V_B = 0$$

$$V_A - 5(1.33) - 5 - 15 + 6(1.5) - V_B = 0$$

$$V_A - V_B = 17.65$$

$$V_{AB} = 17.65 \text{ V}$$

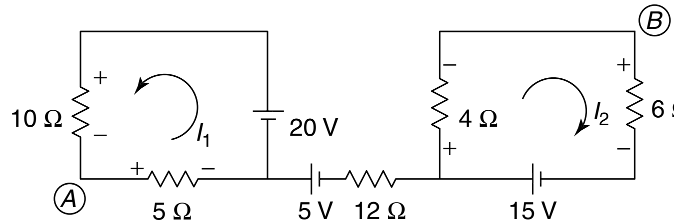

Fig. 2.13

Example 2.7 *Determine the potential difference V_{AB} for the given network in Fig. 2.14.*

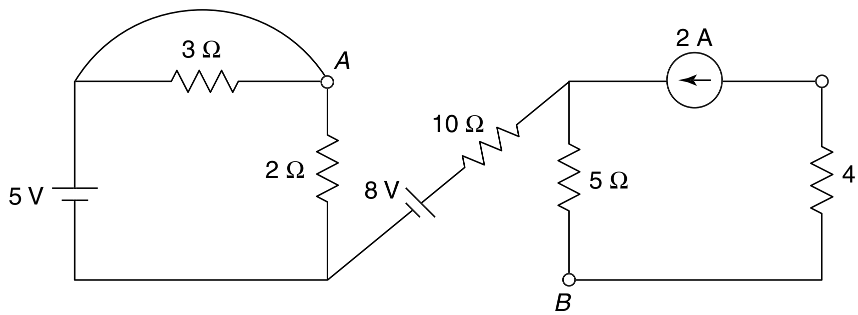

Fig. 2.14

Solution The resistor of 3 Ω is connected across a short circuit. Hence, it gets shorted (Fig. 2.15).

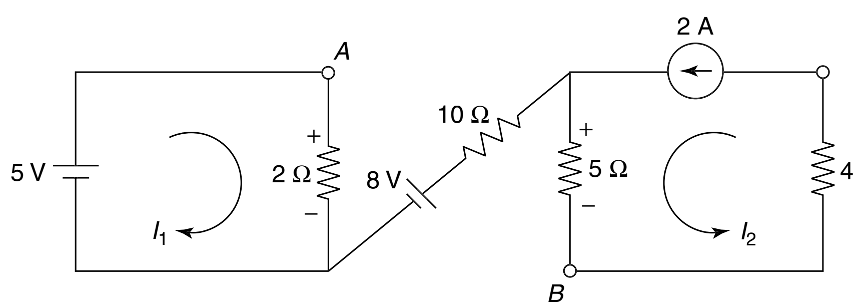

Fig. 2.15

$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Potential difference $\quad V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 8 - 5I_2 - V_B = 0$$

$$V_A - 2(2.5) + 8 - 5(2) - V_B = 0$$

$$V_A - V_B = 7$$
$$V_{AB} = 7\,\text{V}$$

Example 2.8 *Find the voltage of the point A w.r.t. B in Fig. 2.16.*

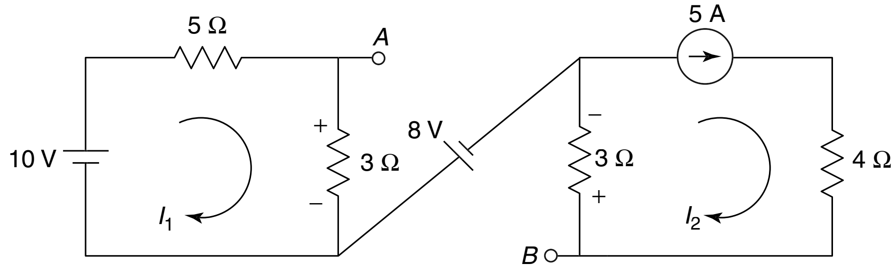

Fig. 2.16

Solution

$$I_1 = \frac{10}{5+3} = 1.25\text{ A}$$
$$I_2 = 5\text{ A}$$

Applying KVL to the path from A to B,

$$V_A - 3I_1 - 8 + 3I_2 - V_B = 0$$
$$V_A - 3(1.25) - 8 + 3(5) - V_B = 0$$
$$V_A - V_B = -3.25$$
$$V_{AB} = -3.25\text{ V}$$

Example 2.9 *In Fig. 2.17, what values must R_1 and R_2 have*

(a) *when $I_1 = 4$ A and $I_2 = 6$ A both charging?*
(b) *when $I_1 = 2$ A discharging and $I_2 = 20$ A charging?*
(c) *when $I_1 = 0$?*

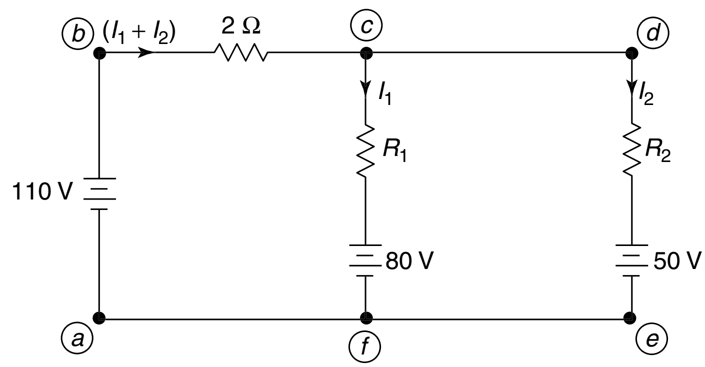

Fig. 2.17

Solution Applying KVL to the closed path *abcfa*,

$$110 - 2(I_1 + I_2) - R_1 I_1 - 80 = 0$$
$$110 - 2I_1 - 2I_2 - R_1 I_1 - 80 = 0$$
$$(2 + R_1)I_1 + 2I_2 = 30 \qquad \ldots\text{(i)}$$

Applying KVL to the closed path *fcdef*,

$$80 + R_1 I_1 - R_2 I_2 - 50 = 0$$
$$R_1 I_1 - R_2 I_2 = -30 \qquad \ldots\text{(ii)}$$

Case (a) $I_1 = 4$ A and $I_2 = 6$ A both charging

i.e. $I_1 = 4$ A and $I_2 = 6$ A

Substituting I_1 and I_2 in Eq. (i),

$$(2 + R_1)\,4 + 2(6) = 30$$
$$R_1 = 2.5\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (ii),

$$2.5(4) - R_2(6) = -30$$
$$R_2 = 6.67\ \Omega$$

Case (b) $I_1 = 2$ A discharging and $I_2 = 20$ A charging

i.e. $I_1 = -2$ A and $I_2 = 20$ A

Substituting I_1 and I_2 in Eq. (i),

$$(2 + R_1)(-2) + 2(20) = 30$$
$$R_1 = 3\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (ii),

$$3(-2) - R_2(20) = -30$$
$$R_2 = 1.2\ \Omega$$

Case (c) $I_1 = 0$

Substituting in Eq. (i)

$$(2 + R_1)(0) + 2I_2 = 30$$
$$I_2 = 15\text{ A}$$

Substituting I_1 and I_2 in Eq. (ii),

$$0 - 15R_2 = -30$$
$$R_2 = 2\ \Omega$$

Example 2.10 *In Fig. 2.18, find I_1 and I_2 when (a) $R = 2.3\ \Omega$, (b) $R = 0.5\ \Omega$, and (c) for what values of R is $I_1 = 0$?*

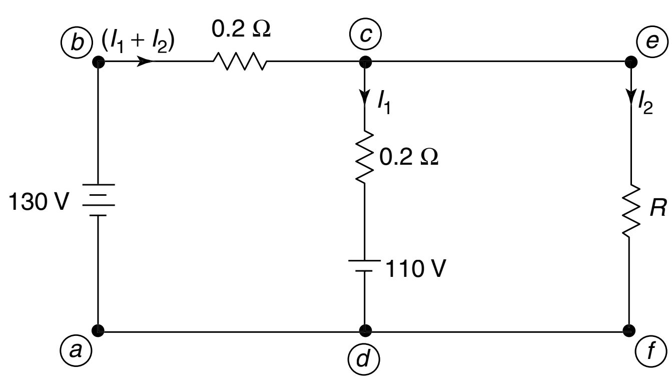

Fig. 2.18

Solution Applying KVL to closed path *abcda*,

$$130 - 0.2(I_1 + I_2) - 0.2I_1 - 110 = 0$$
$$0.4I_1 + 0.2I_2 = 20 \qquad \text{...(i)}$$

Applying KVL to the closed path *dcefd*,

$$110 + 0.2I_1 - RI_2 = 0$$
$$0.2I_1 - RI_2 = -110 \qquad \text{...(ii)}$$

Case (a) $R = 2.3\ \Omega$

Substituting R in Eq. (ii),

$$0.2I_1 - 2.3I_2 = -110 \qquad \text{...(iii)}$$

Solving Eqs (i) and (iii),

$$I_1 = 25\ \text{A}$$
$$I_2 = 50\ \text{A}$$

Case (b) $R = 0.5\ \Omega$

Substituting R in Eq. (i),

$$0.2I_1 - 0.5I_2 = -110 \qquad \text{...(iv)}$$

Solving Eqs (i) and (iv),

$$I_1 = -50\ \text{A}$$
$$I_2 = 200\ \text{A}$$

Case (c) $I_1 = 0$

Substituting I_1 in Eq. (i),

$$0.2I_2 = 20$$
$$I_2 = 100\ \text{A}$$

Substituting I_1 and I_2 in Eq. (ii),

$$0.2(0) - R(100) = -110$$
$$R = 1.1\ \Omega$$

Example 2.11 *In Fig. 2.19, find the value of R.*

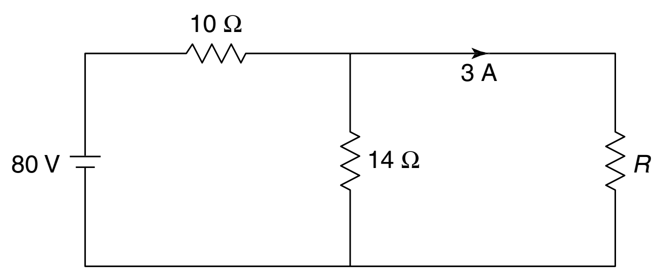

Fig. 2.19

Solution Assigning currents to all the branches (Fig. 2.20),

Applying KVL to the closed path *abcda*,

$$80 - 10I - 14(I - 3) = 0$$
$$I = 5.08\ \text{A}$$

Applying KVL to the closed path *dcefd*,

$$14(I - 3) - 3R = 0$$
$$14(5.08 - 3) - 3R = 0$$
$$R = 9.71\ \Omega$$

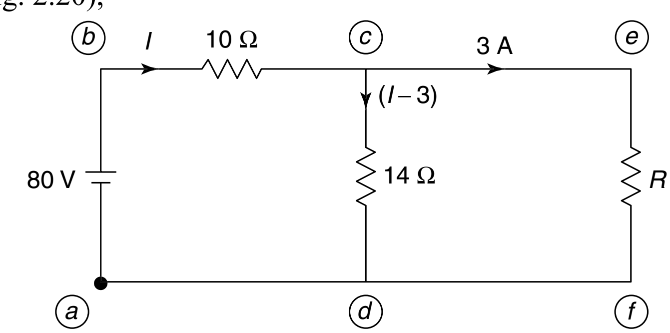

Fig. 2.20

Example 2.12

Determine current drawn by the ammeter shown in Fig. 2.21.

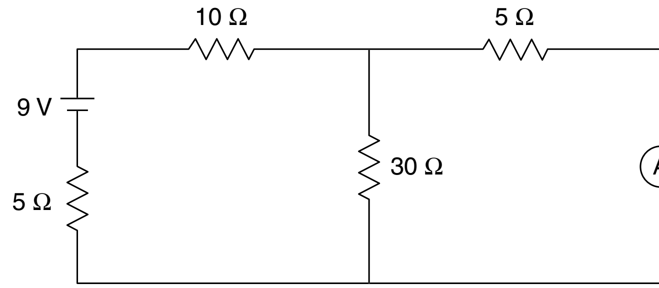

Fig. 2.21

Solution Assigning currents to all the branches (Fig. 2.22),

Applying KVL to the closed path *abcda*,

$$-5(I_1+I_2)+9-10(I_1+I_2)-30I_1=0$$

$$45I_1+15I_2=9 \quad \ldots\text{(i)}$$

Applying KVL to the closed path *dcefd*,

$$30I_1-5I_2=0 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_2=0.4\text{ A}$$

Current drawn by ammeter = 0.4 A

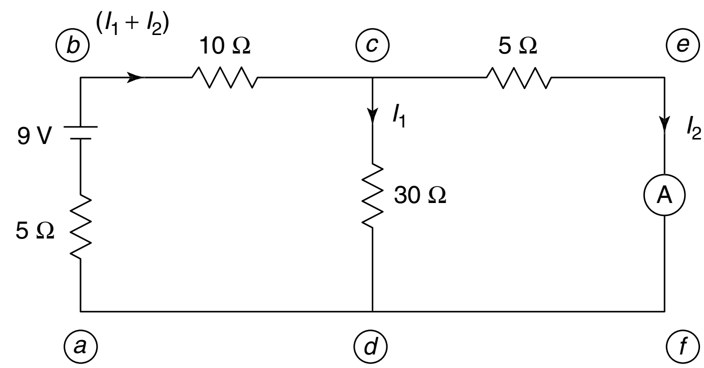

Fig. 2.22

Example 2.13

Find branch currents in the various branches of Fig. 2.23.

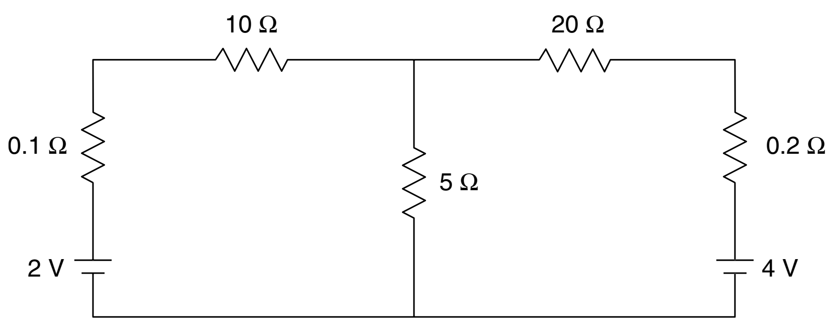

Fig. 2.23

Solution Assigning currents to various branches (Fig. 2.24),

Applying KVL to the closed path *abcda*,

$$2-0.1I_1-10I_1-5(I_1-I_2)=0$$

$$15.1I_1-5I_2=2 \quad \ldots\text{(i)}$$

Applying KVL to the closed path *dcefd*,

$$5(I_1-I_2)-20I_2-0.2I_2-4=0$$

$$5I_1-25.2I_2=4 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1=0.086\text{ A}$$

$$I_2=-0.142\text{ A}$$

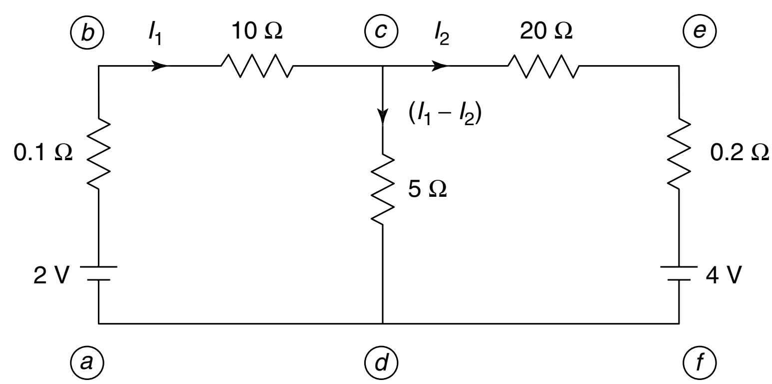

Fig. 2.24

Example 2.14 *In Fig. 2.25, find the value of R and current flowing through it when the current is zero in the branch OA.*

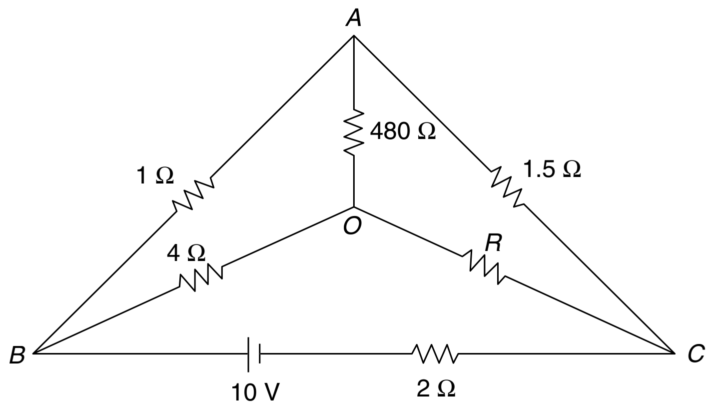

Fig. 2.25

Solution Assigning currents to all branches (Fig. 2.26),
Applying KVL to the closed path *OACO*,

$$480\,I_3 - 1.5(I_1 - I_3) + R(I_2 + I_3) = 0$$

But current in the branch *OA* is zero,
i.e.

$$I_3 = 0$$

$$-1.5\,I_1 + R\,I_2 = 0 \qquad \ldots\text{(i)}$$

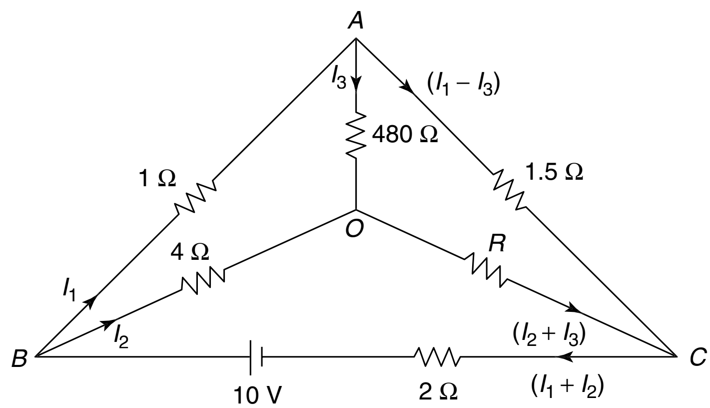

Fig. 2.26

Applying KVL to the closed path *BOCB*,

$$-4\,I_2 - R(I_2 + I_3) - 2(I_1 + I_2) + 10 = 0$$

But $I_3 = 0$

$$-2\,I_1 - (6 + R)\,I_2 = -10 \qquad \ldots\text{(ii)}$$

Applying KVL to the closed path *BOAB*,

$$-4\,I_2 + 480\,I_3 + I_1 = 0$$

But $I_3 = 0$

$$-4\,I_2 + I_1 = 0$$

$$I_1 = 4\,I_2 \qquad \ldots\text{(iii)}$$

Substituting I_1 in Eq. (i) and (ii),

$$-6\,I_2 + R\,I_2 = 0 \qquad \ldots\text{(iv)}$$

and

$$-14\,I_2 - R\,I_2 = -10 \qquad \ldots\text{(v)}$$

From Eq. (iv) and (v),

$$I_2 = 0.5 \text{ A}$$

Substituting I_2 in Eq. (iv),

$$-6(0.5) + R(0.5) = 0$$

$$R = 6\ \Omega$$

Current in branch $OC = I_2 + I_3 = 0.5$ A

Example 2.15 *In Fig. 2.27, find the current supplied by the battery.*

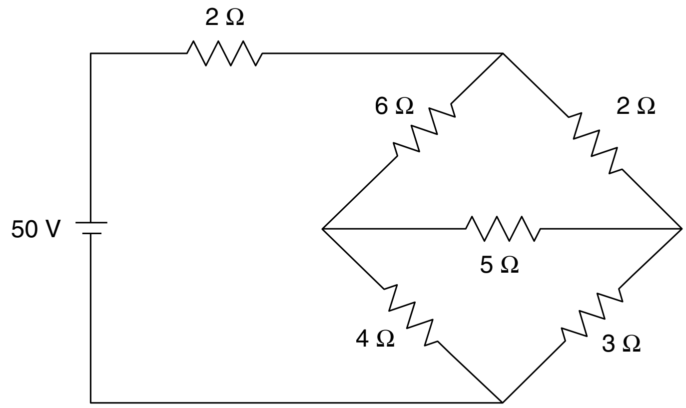

Fig. 2.27

Solution Assigning currents to all the branches (Fig. 2.28),

Applying KVL to the closed path *OABEDO*,

$$50 - 2(I_1 + I_2) - 6I_1 - 4(I_1 - I_3) = 0$$

$$12I_1 + 2I_2 - 4I_3 = 50 \qquad \ldots\text{(i)}$$

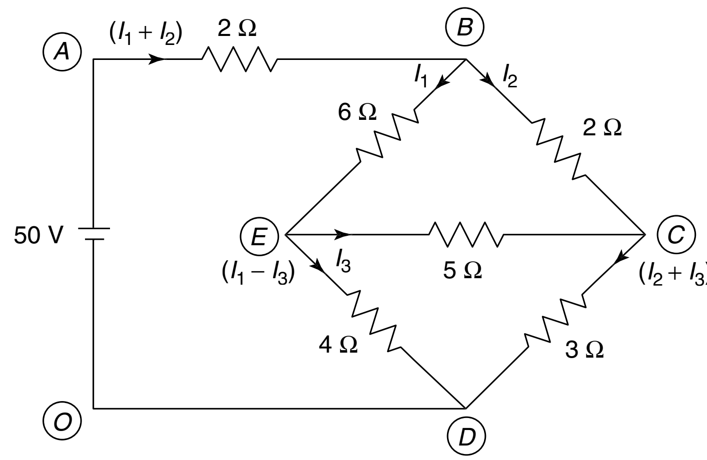

Fig. 2.28

Applying KVL to the closed path *BCEB*,

$$-2I_2 + 5I_3 + 6I_1 = 0$$

$$6I_1 - 2I_2 + 5I_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the closed path *ECDE*,

$$-5I_3 - 3(I_2 + I_3) + 4(I_1 - I_3) = 0$$

$$4I_1 - 3I_2 - 12I_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 2.817 \text{ A}$$

$$I_2 = 6.647 \text{ A}$$

$$I_3 = -0.723 \text{ A}$$

Current supplied by the battery $= I_1 + I_2 = 2.817 + 6.647 = 9.464$ A

Example 2.16 *In Fig.2.29, find the current flowing through the 2 Ω resistor.*

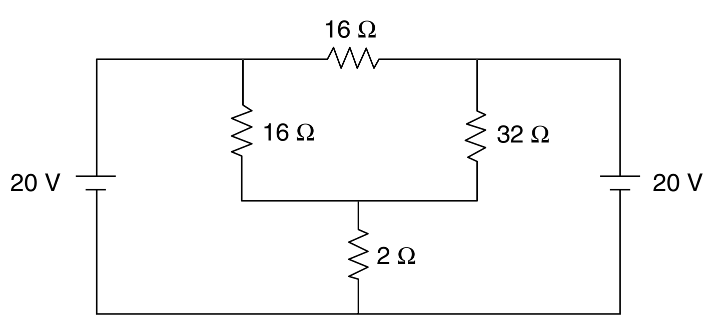

Fig. 2.29

Solution Assigning currents to all the branches (Fig. 2.30),
Applying KVL to the closed path *OABFGO*,

$$20-16I_1-2I_3=0$$
$$16I_1+2I_3=20 \qquad \dots(\text{i})$$

Applying KVL to the closed path *BCFB*,

$$-16I_2+32(I_1-I_3)+16I_1=0$$
$$48I_1-16I_1+32I_3=0 \qquad \dots(\text{ii})$$

Applying KVL to the closed path *GFCDEG*,

$$2I_3-32(I_1-I_3)-20=0$$
$$-32I_1+34I_3=20 \qquad \dots(\text{iii})$$

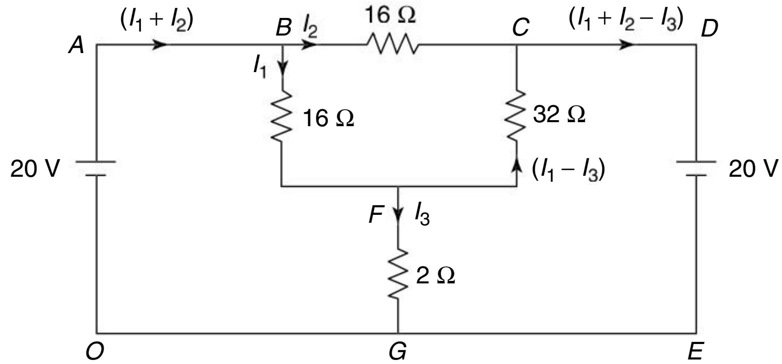

Fig. 2.30

Solving Eqs (i), (ii) and (iii),

$$I_1=1.05 \text{ A}$$
$$I_2=6.32 \text{ A}$$
$$I_3=1.58 \text{ A}$$

Current through the 2 Ω resistor = $I_3=1.58$ A

Example 2.17

In Fig. 2.31, find the current through the 4 Ω resistor.

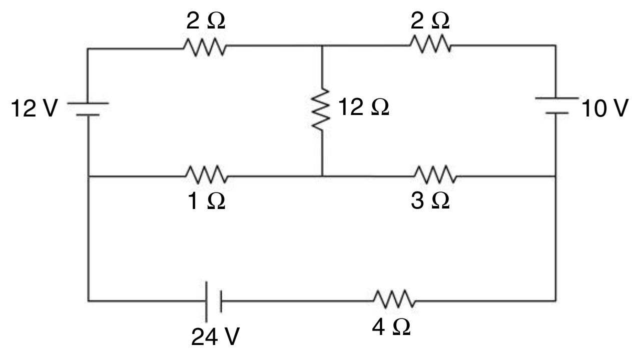

Fig. 2.31

Solution Assigning currents to all the branches (Fig. 2.32),
Applying KVL to the closed path *ABEDA*,

$$-2I_2-12(I_2-I_3)+1(I_1-I_2)+12=0$$
$$I_1-15I_2+12I_3=-12 \quad \dots(\text{i})$$

Applying KVL to the closed path *BCFEB*,

$$-2I_3-10+3(I_1-I_3)+12(I_2-I_3)=0$$
$$3I_1+12I_2-17I_3=10 \quad \dots \ (\text{ii})$$

Applying KVL to the closed path *DEFHGD*,

$$-1(I_1-I_2)-3(I_1-I_3)-4I_1+24=0$$
$$-8I_1+I_2+3I_3=-24 \quad \dots(\text{iii})$$

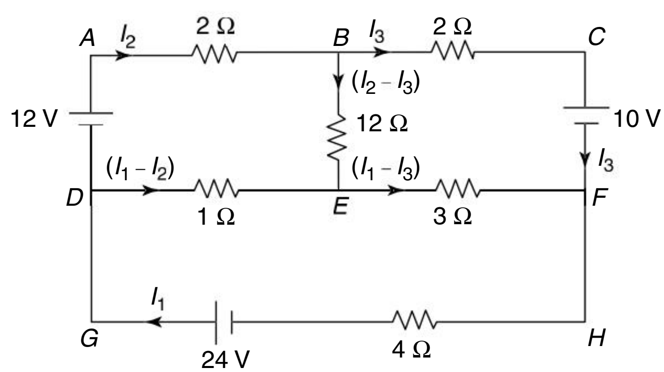

Fig. 2.32

Solving Eqs (i), (ii) and (iii),

$$I_1 = 4.11 \text{ A}$$
$$I_1 = 2.72 \text{ A}$$
$$I_3 = 2.06 \text{ A}$$

Current through the 4 Ω resistor = $I_1 = 4.11$ A

Example 2.18 *In Fig. 2.33, find the current through the 10 Ω resistor.*

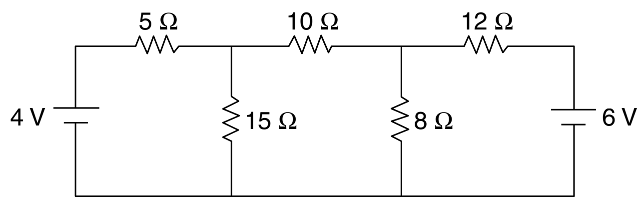

Fig. 2.33

Solution Assigning currents to all the branches (Fig. 2.34),

Applying KVL to the closed path *ABGHA*,

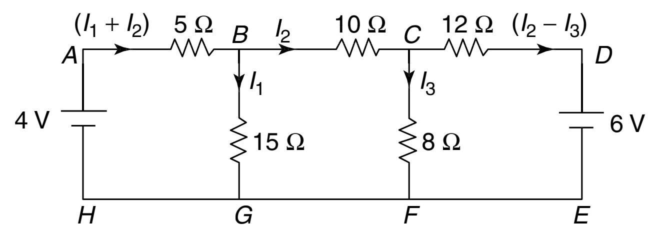

Fig. 2.34

$$-5(I_1 + I_2) - 15I_1 + 4 = 0$$
$$-20I_1 - 5I_2 = -4 \qquad \text{...(i)}$$

Applying KVL to the closed path *BCFGB*,

$$-10I_2 - 8I_3 + 15I_1 = 0$$
$$15I_1 - 10I_2 - 8I_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to the closed path *CDEFC*,

$$-12(I_2 - I_3) - 6 + 8I_3 = 0$$
$$-12I_2 + 20I_3 = 6 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.19 \text{ A}$$
$$I_2 = 0.032 \text{ A}$$
$$I_3 = 0.32 \text{ A}$$

Current through the 10 Ω resistor = $I_2 = 0.032$ A

Example 2.19 *In Fig. 2.35, determine the current supplied by each battery.*

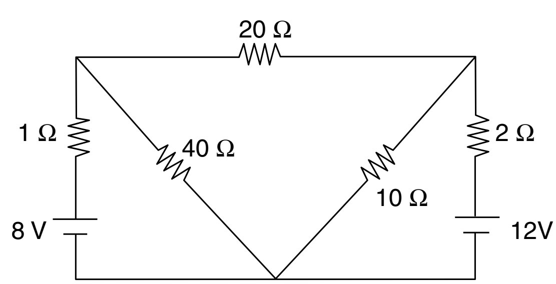

Fig. 2.35

Solution Assigning currents to all the branches (Fig. 2.36),
Applying KVL to the closed path $ADEA$,

$$-40I_2 + 8 - 1I_1 = 0$$
$$I_1 + 40I_2 = 8 \qquad \ldots\text{(i)}$$

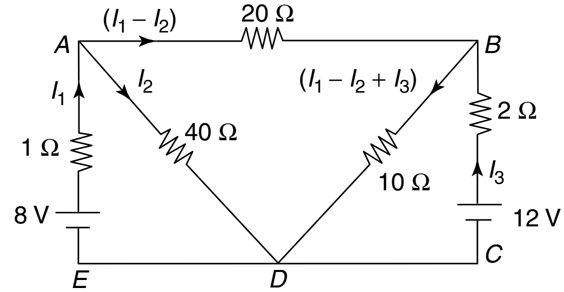

Fig. 2.36

Applying KVL to the closed path $ABDA$,

$$-20(I_1 - I_2) - 10(I_1 - I_2 + I_3) + 40I_2 = 0$$
$$-30I_1 + 70I_2 - 10I_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the closed path $BCDB$,

$$2I_3 - 12 + 10(I_1 - I_2 + I_3) = 0$$
$$10I_1 - 10I_2 + 12I_3 = 12 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$I_1 = 0.1005 \text{ A}$$
$$I_2 = 0.197 \text{ A}$$
$$I_3 = 1.081 \text{ A}$$

Current supplied by the 8 V battery $= I_1 = 0.1005$ A

Current supplied by the 12 V battery $= I_3 = 1.081$ A

Example 2.20 *In Fig. 2.37, find the value of the unknown resistance R such that 2 A current flows through it.*

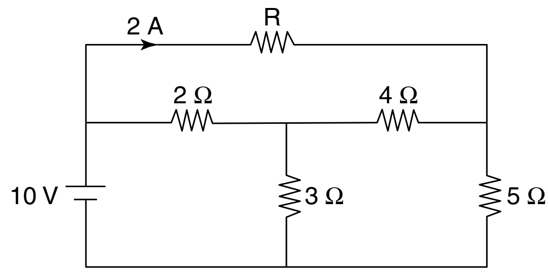

Fig. 2.37

Solution Assigning currents to all the branches (Fig. 2.38),
Applying KVL to the closed path $ABCDEA$,

$$-2R + 4(I_1 - I_2 - 2) + 2(I_1 - 2) = 0$$
$$6I_1 - 4I_2 - 2R = 12 \qquad \ldots\text{(i)}$$

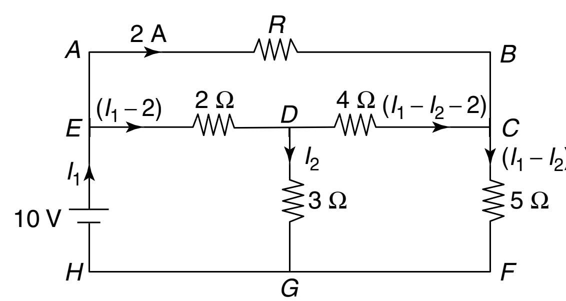

Fig. 2.38

Applying KVL to the closed path $HEDGH$,

$$10 - 2(I_1 - 2) - 3I_2 = 0$$
$$2I_1 + 3I_2 = 14 \qquad \ldots\text{(ii)}$$

Applying KVL to the closed path $GDCFG$,

$$3I_2 - 4(I_1 - I_2 - 2) - 5(I_1 - I_2) = 0$$
$$9I_1 - 12I_2 = 8 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 3.76 \text{ A}$$
$$I_2 = 2.16 \text{ A}$$
$$R = 0.98\ \Omega$$

Unknown resistance $R = 0.98\ \Omega$

Example 2.21 *In Fig. 2.39, find the current delivered by the 12 V battery.*

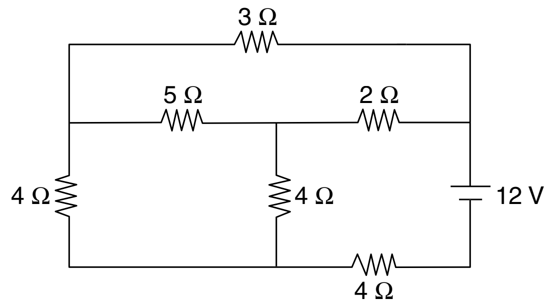

Fig. 2.39

Solution Assigning currents to all the branches (Fig. 2.40),

Applying KVL to the closed path *ABCDEA*,

$$3(I_1 - I_2) - 2I_2 - 5(I_2 - I_3) = 0$$

$$3I_1 - 10I_2 + 5I_3 = 0 \qquad \ldots\text{(i)}$$

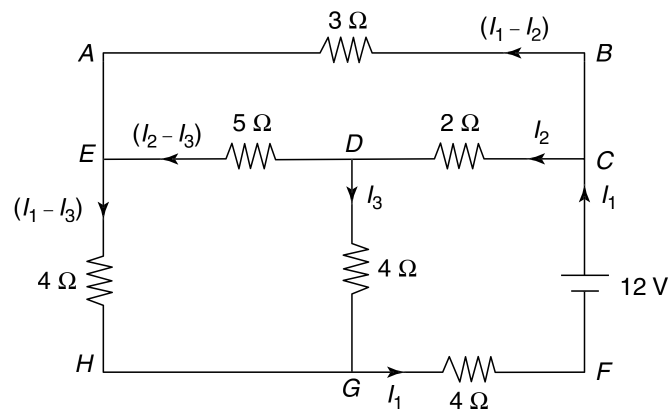

Fig. 2.40

Applying KVL to the closed path *HEDGH*,

$$4(I_1 - I_3) + 5(I_2 - I_3) - 4I_3 = 0$$

$$4I_1 + 5I_2 - 13I_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the closed path *GDCFG*,

$$4I_3 + 2I_2 - 12 + 4I_1 = 0$$

$$4I_1 + 2I_2 + 4I_3 = 12 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$I_1 = 1.66 \text{ A}$$

$$I_2 = 0.93 \text{ A}$$

$$I_3 = 0.87 \text{ A}$$

Current delivered by the 12 V battery $= I_1 = 1.66$ A

EXAMPLES WITH DEPENDENT SOURCES

Example 2.22 *In the network of Fig. 2.41, find I_1, I_2 and V.*

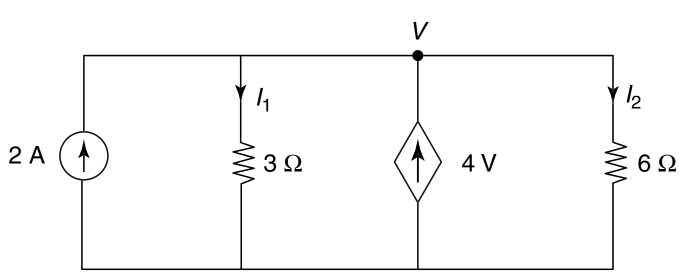

Fig. 2.41

Solution Applying KCL at the node,

$$2 + 4V = \frac{V}{3} + \frac{V}{6}$$

$$\frac{V}{3} + \frac{V}{6} - 4V = 2$$

$$-\frac{7}{2}V = 2$$

$$V = -\frac{4}{7}\text{ V}$$

$$I_1 = \frac{V}{3} = -\frac{4}{21}\text{ A}$$

$$I_2 = \frac{V}{6} = -\frac{2}{21}\text{ A}$$

Example 2.23 *Find voltages V_1 and V_2 in the network of Fig. 2.42.*

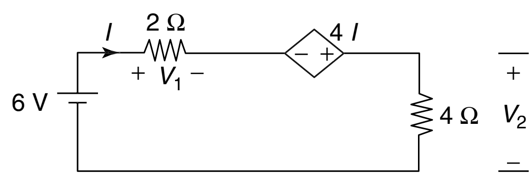

Fig. 2.42

Solution Applying KVL to the loop,

$$6 - 2I + 4I - 4I = 0$$

$$I = 3\text{ A}$$

From Fig. 2.42,

$$V_1 = 2I = 2(3) = 6\text{ V}$$

$$V_2 = 4I = 4(3) = 12\text{ V}$$

Example 2.24 *Find the power delivered by the dependent source in the network of Fig. 2.43.*

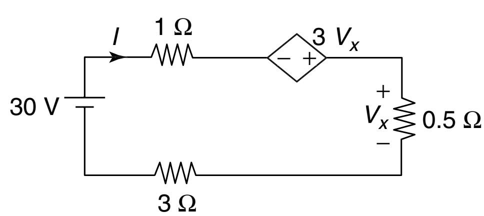

Fig. 2.43

From Fig. 2.43,

$$V_x = 0.5I$$

Applying KVL to the loop,

$$30 - 1I + 3V_x - 0.5I - 3I = 0$$

$$30 - I + 3(0.5I) - 0.5I - 3I = 0$$

$$30 - 3I = 0$$
$$I = 10 \text{ A}$$
$$V_x = 0.5(10) = 5 \text{ V}$$

Power supplied by the dependent source $= (3V_x)(I) = 3 \times 5 \times 10 = 150$ W

Example 2.25 *Find the current I_2 in the network of Fig. 2.44.*

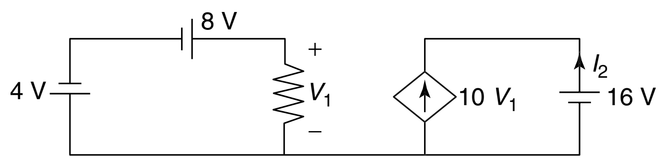

Fig. 2.44

Solution Applying KVL to the left loop,

$$-4 + 8 - V_1 = 0$$
$$V_1 = 4 \text{ V}$$

Applying KCL to the right part,

$$10\,V_1 + I_2 = 0$$
$$10(4) + I_2 = 0$$
$$I_2 = -40 \text{ A}$$

Example 2.26 *Find the voltage V_x in the network of Fig. 2.45.*

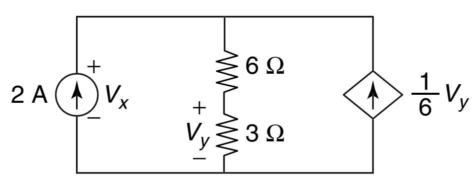

Fig. 2.45

Solution Applying KCL at the node,

$$2 + \frac{1}{6}V_y = \frac{V_x}{9} \qquad \ldots\text{(i)}$$

From Fig. 2.45,

$$V_y = 3\left(\frac{V_x}{9}\right) = \frac{V_x}{3} \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$2 + \frac{1}{6}\left(\frac{V_x}{3}\right) = \frac{V_x}{9}$$
$$2 + \frac{V_x}{18} - \frac{V_x}{9} = 0$$
$$V_x = 36 \text{ V}$$

Example 2.27 *In the network of Fig. 2.46, find the current I_1 and power dissipated in the 500 Ω resistor.*

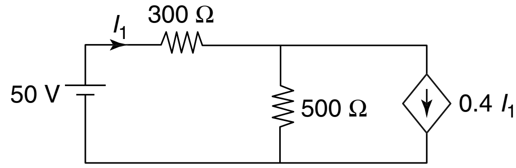

Fig. 2.46

Solution Assigning currents to all the branches as shown in Fig. 2.47.

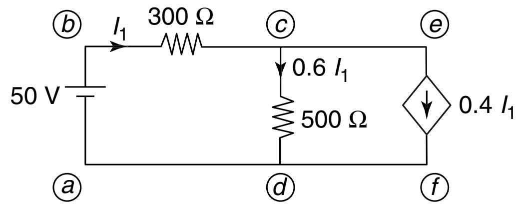

Fig. 2.47

Applying KVL to the closed loop *abcda*,

$$50 - 300\,I_1 - 500(0.6\,I_1) = 0$$

$$I_1 = 0.083 \text{ A}$$

Power dissipated in the 500 Ω resistor $= 500\ (0.6 I_1)^2 = 500\ (0.6 \times 0.083)^2 = 1.24$ W

Example 2.28 *Find the current I in the network shown in Fig. 2.48.*

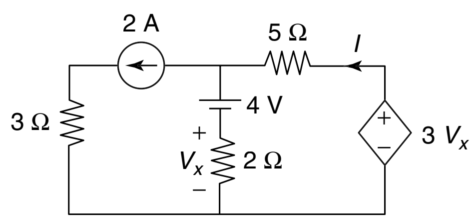

Fig. 2.48

Solution Assigning currents in all the branches as shown in Fig. 2.49.

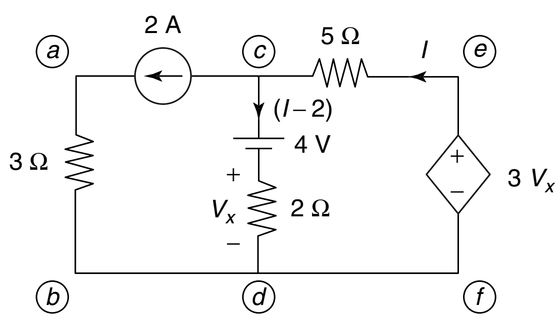

Fig. 2.49

From Fig. 2.49,

$$V_x = 2(I-2) \qquad \text{...(i)}$$

Applying KVL to the closed loop *fecdf*,

$$3V_x - 5I - 4 - 2(I-2) = 0$$

$$3[2(I-2)] - 5I - 4 - 2(I-2) = 0$$

$$6I - 12 - 5I - 4 - 2I + 4 = 0$$
$$-I = 12$$
$$I = -12 \text{ A}$$

2.3 MESH ANALYSIS

A mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applicable only for planar networks. A network is said to be planar if it can be drawn on a plane surface without crossovers. In this method, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. If a network has a large number of voltage sources, it is useful to use mesh analysis. Basically, this analysis consists of writing mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents.

Steps to be Followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

Consider the network shown in Fig. 2.50 which has three meshes. Let the mesh currents for the three meshes be I_1, I_2, and I_3 and all the three mesh currents may be assumed to flow in the clockwise direction. The choice of direction for any mesh current is arbitrary.

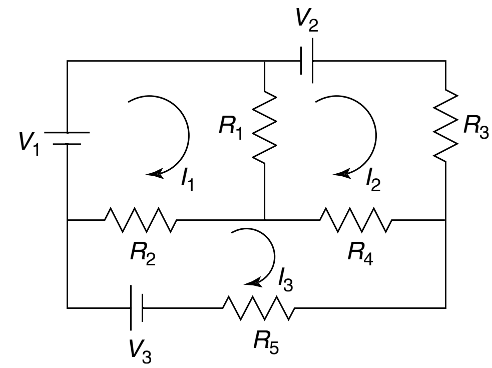

Fig. 2.50

Applying KVL to Mesh 1,

$$V_1 - R_1(I_1 - I_2) - R_2(I_1 - I_3) = 0$$
$$(R_1 + R_2)I_1 - R_1 I_2 - R_2 I_3 = V_1 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - R_3 I_2 - R_4(I_2 - I_3) - R_1(I_2 - I_1) = 0$$
$$-R_1 I_1 + (R_1 + R_3 + R_4)I_2 - R_4 I_3 = V_2 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-R_2(I_3 - I_1) - R_4(I_3 - I_2) - R_5 I_3 + V_3 = 0$$
$$-R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_5)I_3 = V_3 \quad \text{...(iii)}$$

Writing Eqs (i), (ii), and (iii) in matrix form,

$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 \\ -R_1 & R_1 + R_3 + R_4 & -R_4 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

In general,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where, R_{11} = Self-resistance or sum of all the resistance of mesh 1
$R_{12} = R_{21}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 2
$R_{13} = R_{31}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 3
R_{22} = Self-resistance or sum of all the resistance of mesh 2
$R_{23} = R_{32}$ = Mutual resistance or sum of all the resistances common to meshes 2 and 3
R_{33} = Self-resistance or sum of all the resistance of mesh 3

If the directions of the currents passing through the common resistance are the same, the mutual resistance will have a positive sign, and if the direction of the currents passing through common resistance are opposite then the mutual resistance will have a negative sign. If each mesh current is assumed to flow in the clockwise direction then all self-resistances will always be positive and all mutual resistances will always be negative.

The voltages V_1, V_2 and V_3 represent the algebraic sum of all the voltages in meshes 1, 2 and 3 respectively. While going along the current, if we go from negative terminal of the battery to the positive terminal then its emf is taken as positive. Otherwise, it is taken as negative.

Example 2.29 *Find the current through the 5 Ω resistor is shown in Fig. 2.51.*

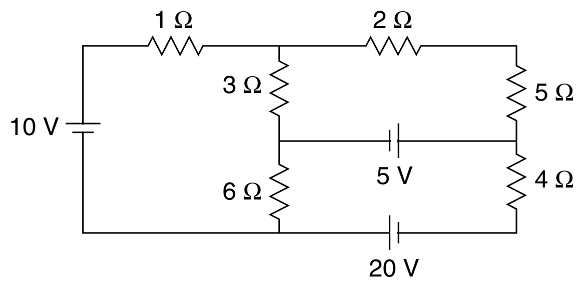

Fig. 2.51

Solution Assigning clockwise currents in three meshes as shown in Fig. 2.52.
Applying KVL to Mesh 1,

$$10 - 1I_1 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$
$$10I_1 - 3I_2 - 6I_3 = 10 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 5I_2 - 5 = 0$$
$$-3I_1 + 10I_2 = -5 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) + 5 - 4I_3 + 20 = 0$$
$$-6I_1 + 10I_3 = 25 \qquad \ldots\text{(iii)}$$

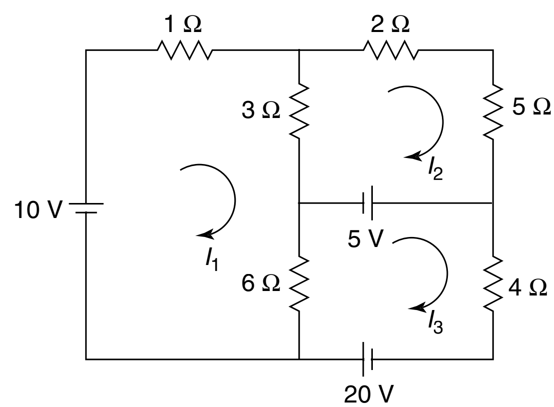

Fig. 2.52

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix}$$

We can write matrix equation directly from Fig. 2.52,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where R_{11} = Self-resistance of Mesh 1 = 1 + 3 + 6 = 10 Ω

R_{12} = Mutual resistance common to meshes 1 and 2 = –3 Ω

Here, negative sign indicates that the current through common resistance are in opposite direction.

R_{13} = Mutual resistance common to meshes 1 and 3 = –6 Ω

Similarly,

$$R_{21} = -3\ \Omega$$
$$R_{22} = 3 + 2 + 5 = 10\ \Omega$$
$$R_{23} = 0$$

$$R_{31} = -6\ \Omega$$
$$R_{32} = 0$$
$$R_{33} = 6 + 4 = 10\ \Omega$$

For voltage matrix,

$$V_1 = 10\text{ V}$$
$$V_2 = -5\text{ V}$$

V_3 = algebraic sum of all the voltages in mesh 3 = 5 + 20 = 25 V

Solving Eqs (i), (ii) and (iii),

$$I_1 = 4.27\text{ A}$$
$$I_2 = 0.78\text{ A}$$
$$I_3 = 5.06\text{ A}$$
$$I_{5\,\Omega} = I_2 = 0.78\text{ A}$$

Example 2.30 *Find the current through the 2 Ω resistor of the network shown in Fig. 2.53.*

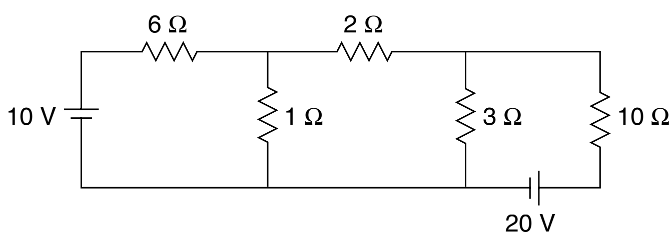

Fig. 2.53

Solution Assigning clockwise currents in three meshes as shown in Fig. 2.54,

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$
$$7I_1 - I_2 = 10 \qquad \text{...(i)}$$

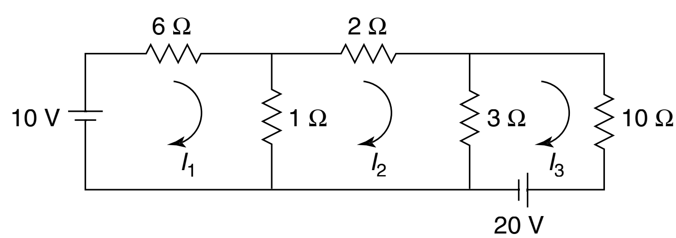

Fig. 2.54

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$
$$-I_1 + 6I_2 - 3I_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 10I_3 - 20 = 0$$
$$-3I_2 + 13I_3 = -20 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 1.34\text{ A}$$
$$I_2 = -0.62\text{ A}$$

$$I_3 = -1.68 \text{ A}$$
$$I_{2\Omega} = I_2 = -0.62 \text{ A}$$

Example 2.31 *Determine the current through the 5 Ω resistor of the network shown in Fig. 2.55.*

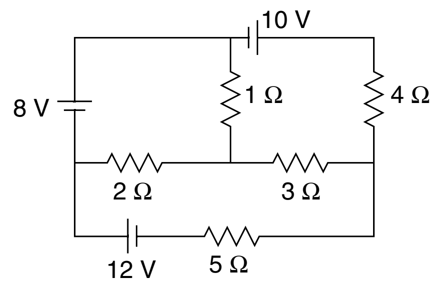

Fig. 2.55

Solution Assigning clockwise currents in three meshes as shown in Fig. 2.56.

Applying KVL to Mesh 1,

$$8 - 1(I_1 - I_2) - 2(I_1 - I_3) = 0$$
$$3I_1 - I_2 - 2I_3 = 8 \qquad \ldots\text{(i)}$$

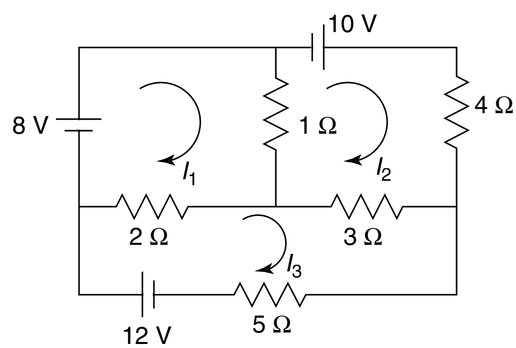

Fig. 2.56

Applying KVL to Mesh 2,

$$10 - 4I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$
$$-I_1 + 8I_2 - 3I_3 = 10 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 3(I_3 - I_2) - 5I_3 + 12 = 0$$
$$-2I_1 - 3I_2 + 10I_3 = 12 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$I_1 = 6.01 \text{ A}$$
$$I_2 = 3.27 \text{ A}$$
$$I_3 = 3.38 \text{ A}$$
$$I_{5\Omega} = I_3 = 3.38 \text{ A}$$

Example 2.32 *Find the current supplied by the battery of the network shown in Fig. 2.57.*

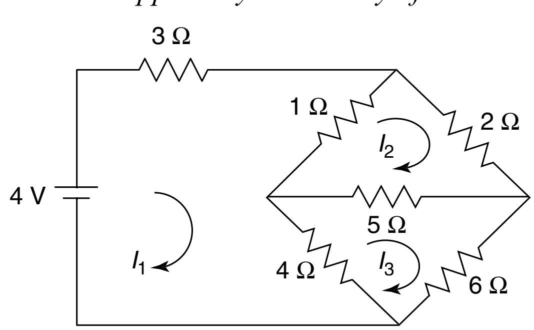

Fig. 2.57

Solution

Applying KVL to Mesh 1,

$$4-3I_1-1(I_1-I_2)-4(I_1-I_3)=0$$

$$8I_1-I_2-4I_3=4 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-2I_2-5(I_2-I_3)-1(I_2-I_1)=0$$

$$-I_1+8I_2-5I_3=0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6I_3-4(I_3-I_1)-5(I_3-I_2)=0$$

$$-4I_1-5I_2+15I_3=0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=0.66 \text{ A}$$

$$I_2=0.24 \text{ A}$$

$$I_3=0.26 \text{ A}$$

Current supplied by the battery $= I_1 = 0.66$ A.

Example 2.33 *Find the current through the 4 Ω resistor in the network of Fig. 2.58.*

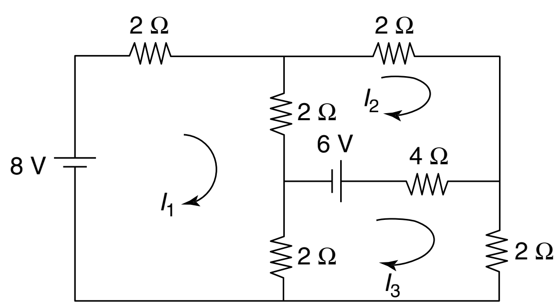

Fig. 2.58

Solution Applying KVL to Mesh 1,

$$8-2I_1-2(I_1-I_2)-2(I_1-I_3)=0$$

$$6I_1-2I_2-2I_3=8 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-2(I_2-I_1)-2I_2-4(I_2-I_3)-6=0$$

$$-2I_1+8I_2-4I_3=-6 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3-I_1)+6-4(I_3-I_2)-2I_3=0$$

$$-2I_1-4I_2+8I_3=6 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=2 \text{ A}$$

$$I_2=0.5 \text{ A}$$

$$I_3=1.5 \text{ A}$$

$$I_{4\Omega}=I_3-I_2=1.5-0.5=1 \text{ A}$$

Example 2.34 *Determine the voltage V which causes the current I_1 to be zero in the network of Fig. 2.59.*

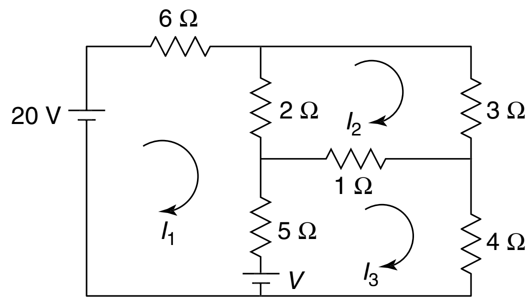

Fig. 2.59

Solution Applying KVL to Mesh 1,

$$20-6I_1-2(I_1-I_2)-5(I_1-I_3)-V=0$$

$$V+13I_1-2I_2-5I_3=20 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-2(I_2-I_1)-3I_2-1(I_2-I_3)=0$$

$$2I_1-6I_2+I_3=0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-1(I_3-I_2)-4I_3+V-5(I_3-I_1)=0$$

$$V+5I_1+I_2-10I_3=0 \qquad \ldots\text{(iii)}$$

Putting $I_1=0$ in Eqs (i), (ii) and (iii),

$$V-2I_2-5I_3=20$$

$$-6I_2+I_3=0$$

$$V+I_2-10I_3=0 \qquad \ldots\text{(iv)}$$

Solving Eqs (i), (ii) and (iii),

$$V=43.7\text{ V}$$

Example 2.35 *Find the current through the 2 Ω resistor in the network of Fig. 2.60.*

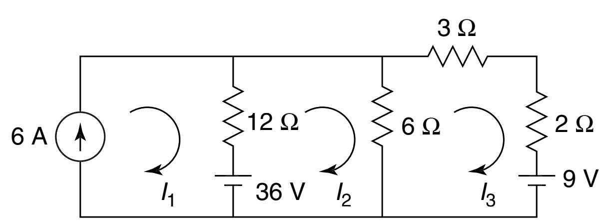

Fig. 2.60

Solution Mesh 1 contains a current source of 6 A. Hence, we can write current equation for mesh 1. Since direction of current source and mesh current I_1 are same,

$$I_1=6 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$36-12(I_2-I_1)-6(I_2-I_3)=0$$
$$36-12(I_2-6)-6I_2+6I_3=0$$
$$18I_2-6I_3=108 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3-I_2)-3I_3-2I_3-9=0$$
$$6I_2-11I_3=9 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_3=3\text{ A}$$
$$I_{2\,\Omega}=I_3=3\text{ A}$$

Example 2.36 *Determine the mesh currents I_1, I_2 and I_3 in the network of Fig. 2.61.*

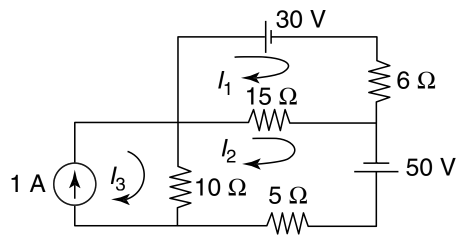

Fig. 2.61

Solution Applying KVL to Mesh 1,

$$-30-6I_1-15(I_1-I_2)=0$$
$$21I_1-15I_2=-30 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-10(I_2-I_3)-15(I_2-I_1)+50-5I_2=0$$
$$-15I_1+30I_2-10I_3=50 \qquad \text{...(ii)}$$

For Mesh 3,

$$I_3=1 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=0$$
$$I_2=2\text{ A}$$
$$I_3=1\text{ A}$$

Example 2.37 *Find the current through the 5 Ω resistor in the network of Fig. 2.62.*

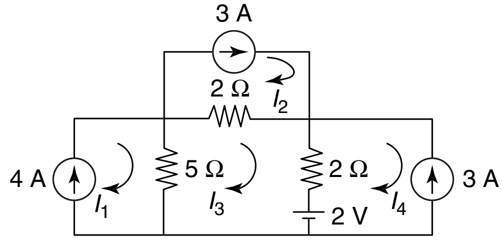

Fig. 2.62

Solution Writing current equations for Meshes 1, 2 and 4,

$$I_1 = 4 \qquad \text{...(i)}$$

$$I_2 = 3 \qquad \text{...(ii)}$$

$$I_4 = -3 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-5(I_3 - I_1) - 2(I_3 - I_2) - 2(I_3 - I_4) - 2 = 0 \qquad \text{...(iv)}$$

Substituting Eqs (i), (ii) and (iii) in Eq. (iv),

$$-5(I_3 - 4) - 2(I_3 - 3) - 2(I_3 + 3) - 2 = 0$$

$$I_3 = 2\text{ A}$$

$$I_{5\Omega} = I_1 - I_3 = 4 - 2 = 2\text{ A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.38 *Obtain the branch currents in the network shown in Fig. 2.63.*

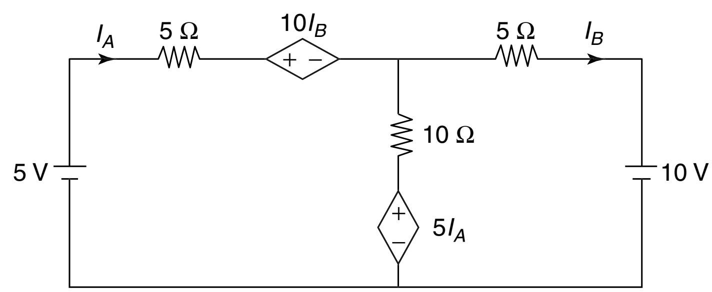

Fig. 2.63

Solution Assigning clockwise currents in two meshes as shown in Fig. 2.64,
From Fig. 2.64,

$$I_A = I_1 \qquad \text{...(i)}$$

$$I_B = I_2 \qquad \text{...(ii)}$$

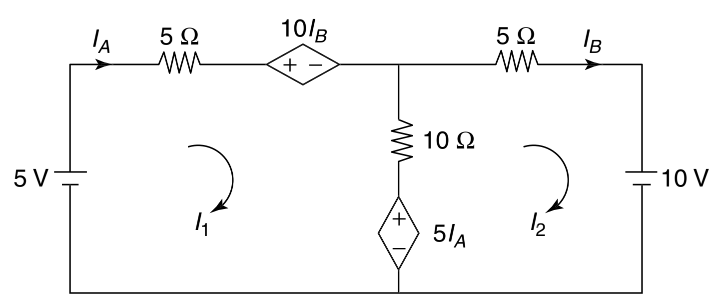

Fig. 2.64

Applying KVL to Mesh 1,

$$5 - 5I_1 - 10I_B - 10(I_1 - I_2) - 5I_A = 0$$

$$5 - 5I_1 - 10I_2 - 10I_1 + 10I_2 - 5I_1 = 0$$

$$-20I_1 = -5$$

$$I_1 = \frac{1}{4} = 0.25\text{ A} \qquad \text{...(iii)}$$

Applying KVL to Mesh 2,

$$5I_A - 10(I_2 - I_1) - 5I_2 - 10 = 0$$

$$5I_1 - 10I_2 + 10I_1 - 5I_2 = 10$$

$$15I_1 - 15I_2 = 10 \qquad \text{...(iv)}$$

Putting $I_1 = 0.25$ A in Eq. (iv),

$$15(0.25) - 15I_2 = 10$$

$$I_2 = -0.416 \text{ A}$$

Example 2.39 *Find the mesh currents in the network shown in Fig. 2.65.*

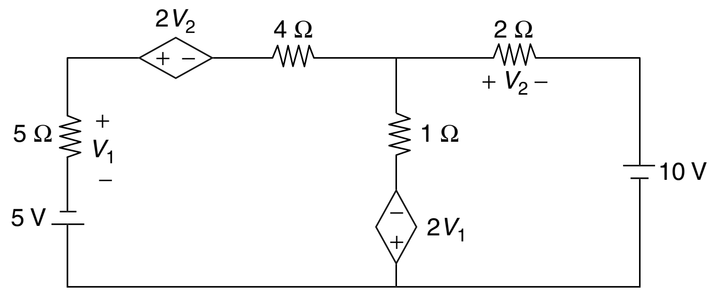

Fig. 2.65

Solution Assigning clockwise currents in the two meshes as shown in Fig. 2.66,

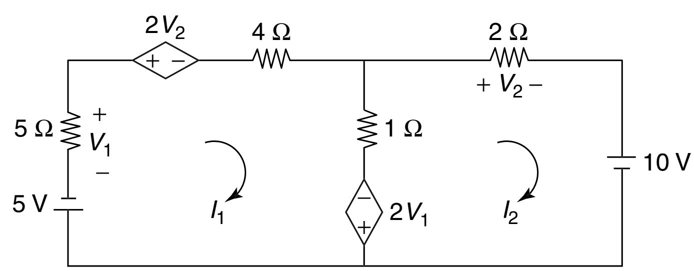

Fig. 2.66

From Fig. 2.66, ...(i)

$$V_1 = -5I_1$$

$$V_2 = 2I_2 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$-5 - 5I_1 - 2V_2 - 4I_1 - 1(I_1 - I_2) + 2V_1 = 0$$

$$-5 - 5I_1 - 2(2I_2) - 4I_1 - I_1 + I_2 + 2(-5I_1) = 0$$

$$20I_1 + 3I_2 = -5 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-2V_1 - 1(I_2 - I_1) - 2I_2 - 10 = 0$$

$$-2(-5I_1) - I_2 + I_1 - 2I_2 = 10$$

$$11I_1 - 3I_2 = 10 \qquad \ldots\text{(iv)}$$

Solving Eqs (iii) and (iv),

$$I_1 = 0.161 \text{ A}$$

$$I_2 = -2.742 \text{ A}$$

Example 2.40 *Find currents I_x and I_y of the network shown in the Fig. 2.67.*

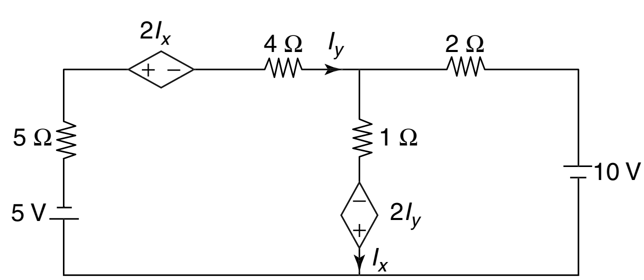

Fig. 2.67

Solution Assigning clockwise currents in the two Meshes as shown in Fig. 2.68.

From Fig. 2.68,

$$I_y = I_1$$

$$I_x = I_1 - I_2$$

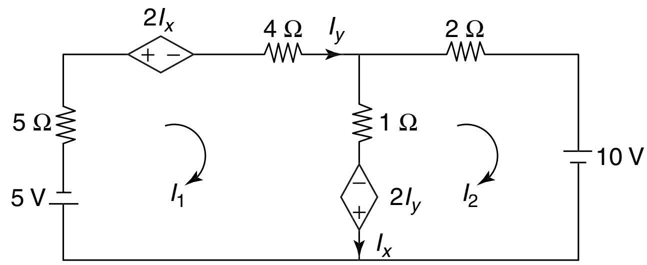

Fig. 2.68

Applying KVL to Mesh 1,

$$-5 - 5I_1 - 2I_x - 4I_1 - 1(I_1 - I_2) + 2I_y = 0$$

$$-5 - 5I_1 - 2(I_1 - I_2) - 4I_1 - I_1 + I_2 + 2I_1 = 0$$

$$-5 - 5I_1 - 2I_1 + 2I_2 - 4I_1 - I_1 + I_2 + 2I_1 = 0$$

$$-10I_1 + 3I_2 = 5 \qquad \text{...(iii)}$$

Applying KVL to Mesh 2,

$$-2I_y - 1(I_2 - I_1) - 2I_2 - 10 = 0$$

$$-2I_1 - I_2 + I_1 - 2I_2 = 10$$

$$-I_1 - 3I_2 = 10 \qquad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$I_1 = -\frac{15}{11} = -1.364 \text{ A}$$

$$I_2 = -2.878 \text{ A}$$

$$I_y = -1.364 \text{ A}$$

$$I_x = I_1 - I_2 = -1.364 + 2.878 = 1.514 \text{ A}$$

Example 2.41 *Find the currents in the three meshes of the network shown in Fig. 2.69.*

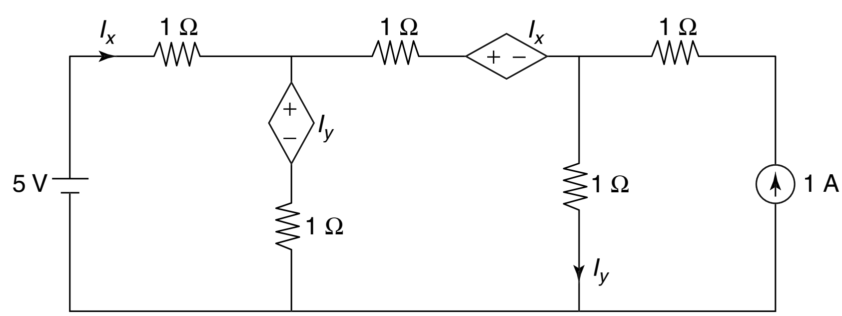

Fig. 2.69

Solution Assigning clockwise currents in the three meshes is shown in Fig. 2.70.

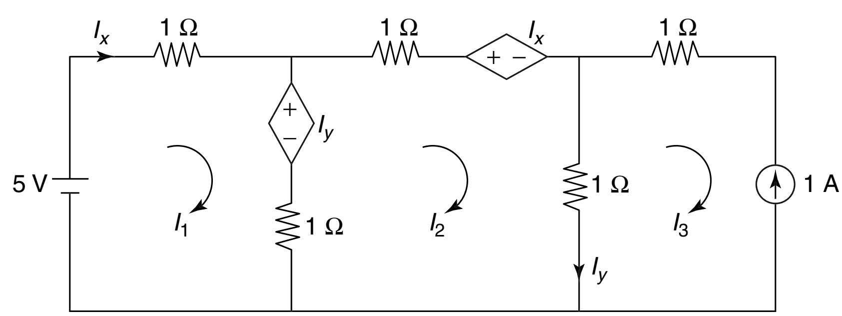

Fig. 2.70

From Fig. 2.70,

$$I_x = I_1 \quad \text{...(i)}$$
$$I_y = I_2 - I_3 \quad \text{...(ii)}$$

Applying KVL to Mesh 1,

$$5 - 1I_1 - I_y - 1(I_1 - I_2) = 0$$
$$5 - I_1 - (I_2 - I_3) - (I_1 - I_2) = 0$$
$$-2I_1 + I_3 = -5 \quad \text{...(iii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + I_y - 1I_2 - I_x - 1(I_2 - I_3) = 0$$
$$-(I_2 - I_1) + (I_2 - I_3) - I_2 - I_1 - (I_2 - I_3) = 0$$
$$-2I_2 = 0 \quad \text{...(iv)}$$

For Mesh 3,

$$I_3 = -1 \quad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = 2 \text{ A}$$
$$I_2 = 0$$
$$I_3 = -1 \text{ A}$$

Example 2.42 *For the network shown in Fig. 2.71, find the power supplied by the dependent voltage source.*

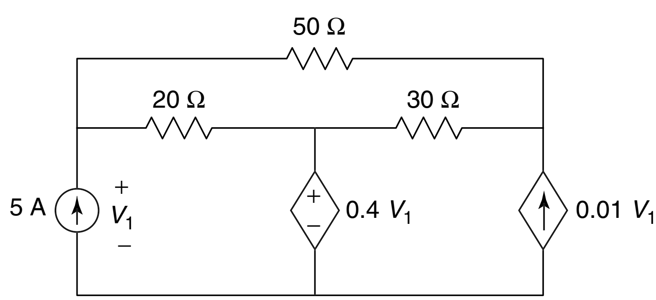

Fig. 2.71

Solution Assigning clockwise currents in three Meshes as shown in Fig. 2.72.

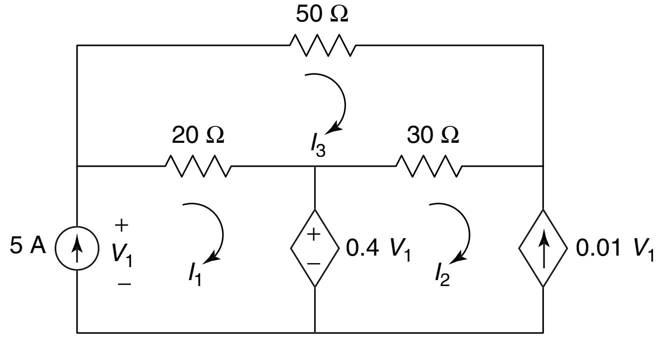

Fig. 2.72

From Fig. 2.72,

$$V_1 - 20(I_1 - I_3) - 0.4\,V_1 = 0$$
$$0.6\,V_1 = 20\,I_1 - 20\,I_3$$
$$V_1 = 33.33\,I_1 - 33.33\,I_3 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 5 \qquad \text{...(ii)}$$

For Mesh 2,

$$I_2 = -0.01\,V_1 = -0.01(33.33\,I_1 - 33.33\,I_3)$$
$$0.33\,I_1 + I_2 - 0.33\,I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-50\,I_3 - 30(I_3 - I_2) - 20(I_3 - I_1) = 0$$
$$-20\,I_1 - 30\,I_2 + 100\,I_3 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 5 \text{ A}$$
$$I_2 = -1.47 \text{ A}$$
$$I_3 = 0.56 \text{ A}$$

$$V_1 = 33.33\,I_1 - 33.33\,I_3 = 33.33(5) - 33.33(0.56) = 148 \text{ V}$$

Power supplied by the dependent voltage source $= 0.4\ V_1(I_1 - I_2) = 0.4\ (148)(5 + 1.47) = 383.02$ W

Example 2.43 *Find the voltage V_x in the network shown in Fig. 2.73.*

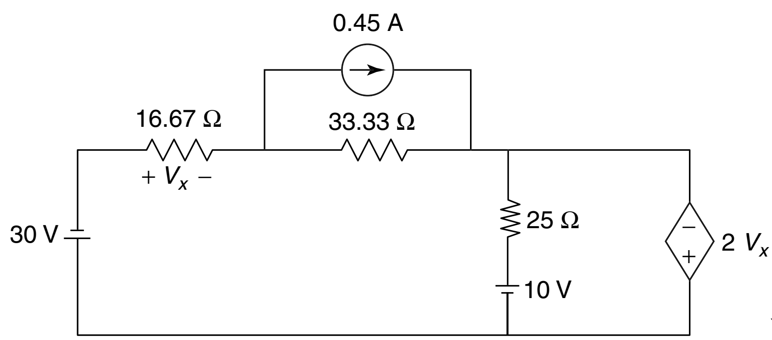

Fig. 2.73

Solution Assigning clockwise currents in the three meshes as shown in Fig. 2.74.

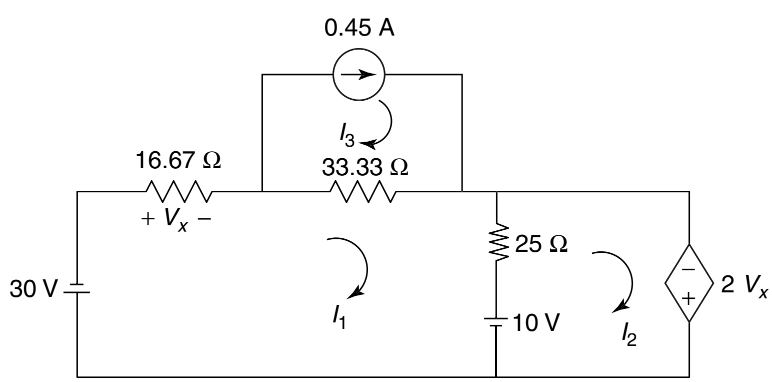

Fig. 2.74

From Fig. 2.74,

$$V_x = 16.67\,I_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$-30 - 16.67\,I_1 - 33.33(I_1 - I_3) - 25(I_1 - I_2) - 10 = 0$$
$$-30 - 16.67\,I_1 - 33.33\,I_1 + 33.33\,I_3 - 25\,I_1 + 25\,I_2 - 10 = 0$$
$$-75\,I_1 + 25\,I_2 + 33.33\,I_3 = 40 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$10 - 25(I_2 - I_1) + 2\,V_x = 0$$
$$10 - 25(I_2 - I_1) + 2(16.67\,I_1) = 0$$
$$10 - 25\,I_2 + 25\,I_1 + 33.34\,I_1 = 0$$
$$58.34\,I_1 - 25\,I_2 = -10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = 0.45 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -0.9\text{ A}$$
$$I_2 = -1.7\text{ A}$$
$$I_3 = 0.45\text{ A}$$
$$V_x = 16.67\,I_1 = 16.67\,(-0.9) = -15\text{ V} \qquad \ldots\text{(v)}$$

Example 2.44 *For the network shown in Fig. 2.75, find the mesh currents I_1, I_2 and I_3.*

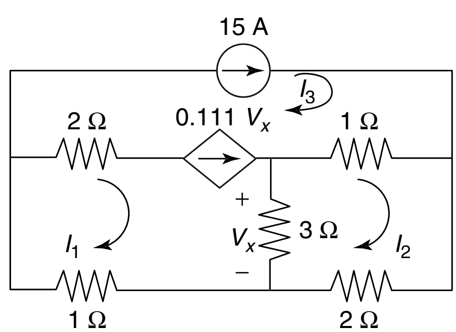

Fig. 2.75

Solution From Fig. 2.75,

$$V_x = 3(I_1 - I_2) \qquad \ldots\text{(i)}$$

Writing current equation for the two current sources,

$$I_3 = 15 \qquad \ldots\text{(ii)}$$

and

$$0.111\,V_x = I_1 - I_3$$
$$0.111\,[3\,(I_1 - I_2) = I_1 - I_3$$
$$0.333\,I_1 - 0.333\,I_2 - I_1 + I_3 = 0$$
$$-0.667\,I_1 - 0.333\,I_2 + I_3 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 1(I_2 - I_3) - 2I_2 = 0$$

$$-3I_1 + 6I_2 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 17 \text{ A}$$

$$I_2 = 11 \text{ A}$$

$$I_3 = 15 \text{ A}$$

Example 2.45 *For the network shown in Fig. 2.76, find the magnitude of V_0 and the current supplied by it, given that power loss in $R_L = 2\ \Omega$ resistor is 18 W.*

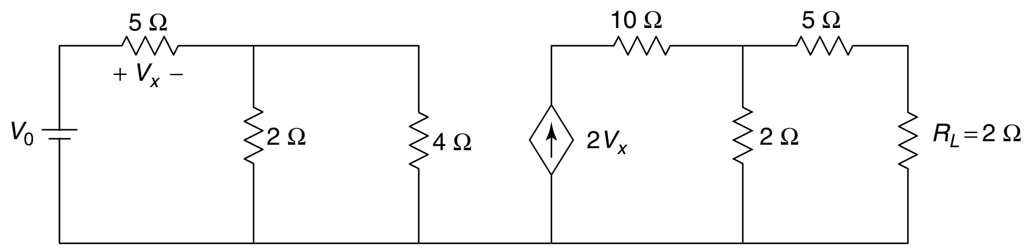

Fig. 2.76

Solution Assigning clockwise currents in meshes is shown in Fig. 2.77.

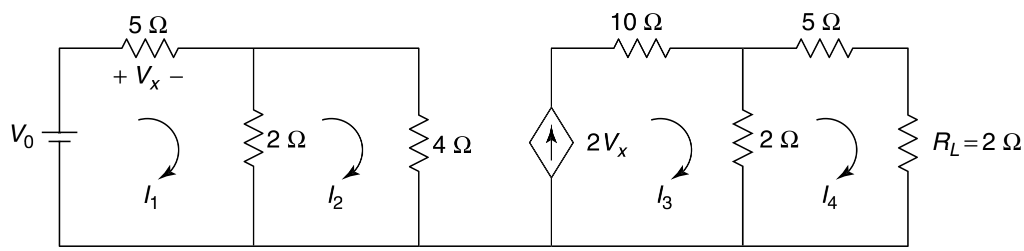

Fig. 2.77

From Fig. 2.77,

$$V_x = 5I_1 \qquad \ldots\text{(i)}$$

Also,

$$I_4^2 R_L = 18$$

$$I_4^2(2) = 18$$

$$I_4 = 3 \text{ A} \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$V_0 - 5I_1 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = V_0 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 4I_2 = 0$$

$$-2I_1 + 6I_2 = 0 \qquad \ldots\text{(iv)}$$

For Mesh 3,

$$I_3 = 2V_x = 2(5I_1) = 10I_1$$

$$10I_1 - I_3 = 0 \qquad \ldots\text{(v)}$$

Applying KVL to Mesh 4,

$$-2(I_4 - I_3) - 5I_4 - 2I_4 = 0$$
$$-2I_3 + 9I_4 = 0$$
$$-2I_3 + 9(3) = 0$$
$$I_3 = 13.5 \text{ A}$$

From Eq. (v),

$$I_1 = \frac{I_3}{10} = \frac{13.5}{10} = 1.35 \text{ A}$$

From Eq. (iv),

$$-2(1.35) + 6I_2 = 0$$
$$I_2 = 0.45 \text{ A}$$

From Eq. (iii),

$$7(1.35) - 2(0.45) = V_0$$
$$V_0 = 8.55 \text{ V}$$

Current supplied by voltage source $V_0 = I_1 = 1.35$ A

Example 2.46 *In the network shown in Fig. 2.78, find voltage V_2 such that $V_x = 0$.*

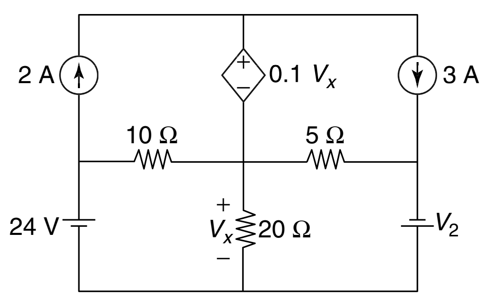

Fig. 2.78

Solution Assigning clockwise currents in four meshes as shown in Fig. 2.79.

From Fig. 2.79,

$$V_x = 20(I_3 - I_4) \qquad \ldots\text{(i)}$$

Writing current equations for Meshes 1 and 2,

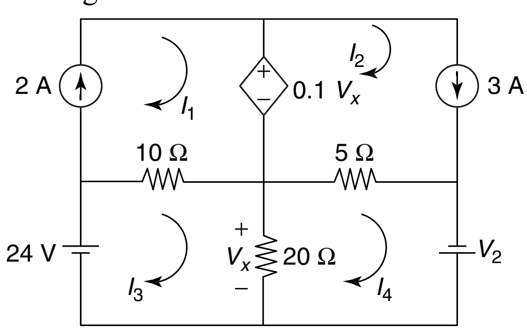

Fig. 2.79

$$I_1 = 2 \qquad \ldots\text{(ii)}$$
$$I_2 = 3 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$24 - 10(I_3 - I_1) - 20(I_3 - I_4) = 0$$
$$24 - 10(I_3 - 2) - 20(I_3 - I_4) = 0$$
$$-30I_3 + 20I_4 = -44 \qquad \ldots\text{(iv)}$$

Applying KVL to Mesh 4,

$$-20(I_4 - I_3) - 5(I_4 - I_2) + V_2 = 0$$
$$-20(I_4 - I_3) - 5(I_4 - 3) + V_2 = 0$$
$$20I_3 - 25I_4 = -V_2 - 15 \qquad \ldots\text{(v)}$$

But

$$V_x = 0$$
$$20(I_3 - I_4) = 0$$
$$I_3 = I_4$$

From Eq. (iv),

$$-30 I_3 + 20 I_3 = -44$$
$$I_3 = 4.4 \text{ A}$$
$$I_4 = 4.4 \text{ A}$$

From Eq. (v),

$$20\,(4.4) - 25\,(4.4) = -V_2 - 15$$
$$V_2 = 7 \text{ V}$$

2.4 SUPERMESH ANALYSIS

Meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a supermesh. A path around a supermesh doesn't pass through a current source. A path around each mesh contained within a supermesh passes through a current source. The total number of equations required for a supermesh is equal to the number of meshes contained in the supermesh. A supermesh requires one mesh current equation, that is, a KVL equation. The remaining mesh current equations are KCL equations.

Example 2.47 *Find the current through the 10 Ω resistor of the network shown in Fig. 2.80.*

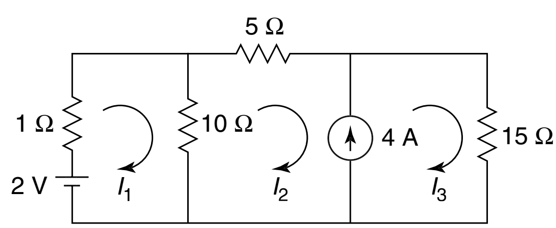

Fig. 2.80

Solution Applying KVL to Mesh 1,

$$2 - 1 I_1 - 10 (I_1 - I_2) = 0$$
$$11 I_1 - 10 I_2 = 2 \qquad \text{...(i)}$$

Since meshes 2 and 3 contain a current source of 4 A, these two meshes will form a supermesh. A supermesh is formed by two adjacent meshes that have a common current source. The direction of the current source of 4 A and current $(I_3 - I_2)$ are same, i.e., in the upward direction.
Writing current equation to the supermesh,1

$$I_3 - I_2 = 4 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10 (I_2 - I_1) - 5 I_2 - 15 I_3 = 0$$
$$10 I_1 - 15 I_2 - 15 I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = -2.35 \text{ A}$$
$$I_2 = -2.78 \text{ A}$$
$$I_3 = 1.22 \text{ A} \qquad \text{...(iv)}$$

Current through the 10 Ω resistor $= I_1 - I_2 = -(2.35) - (-2.78) = 0.43$ A

Example 2.48 *Find the current in the 3 Ω resistor of the network shown in Fig. 2.81.*

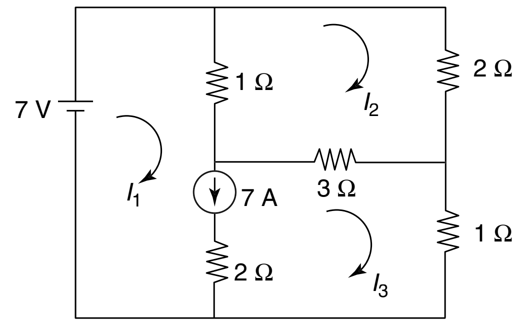

Fig. 2.81

Solution Meshes 1 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_3 = 7 \qquad \text{...(i)}$$

Applying KVL to the outer path of the supermesh,

$$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - 1I_3 = 0$$
$$-I_1 + 4I_2 - 4I_3 = -7 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$
$$I_1 - 6I_2 + 3I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 9 \text{ A}$$
$$I_2 = 2.5 \text{ A}$$
$$I_3 = 2 \text{ A}$$

Current through the 3 Ω resistor $= I_2 - I_3 = 2.5 - 2 = 0.5$ A

Example 2.49 *Find the current in the 5 Ω resistor of the network shown in Fig. 2.82.*

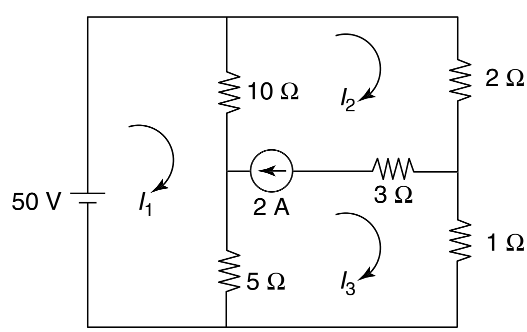

Fig. 2.82

Solution Applying KVL to Mesh 1,

$$50-10(I_1-I_2)-5(I_1-I_3)=0$$

$$15I_1-10I_2-5I_3=50 \qquad \text{...(i)}$$

Meshes 2 and 3 will form a supermesh as these two meshes share a common current source of 2 A. Writing current equation for the supermesh,

$$I_2-I_3=2 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10(I_2-I_1)-2I_2-1I_3-5(I_3-I_1)=0$$

$$-15I_1+12I_2+6I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=20 \text{ A}$$

$$I_2=17.33 \text{ A}$$

$$I_3=15.33 \text{ A}$$

Current through the 5 Ω resistor = $I_1 - I_3$ = 20 – 15.33 = 4.67 A

Example 2.50 *Determine the power delivered by the voltage source and the current in the 10 Ω resistor of the network shown in Fig. 2.83.*

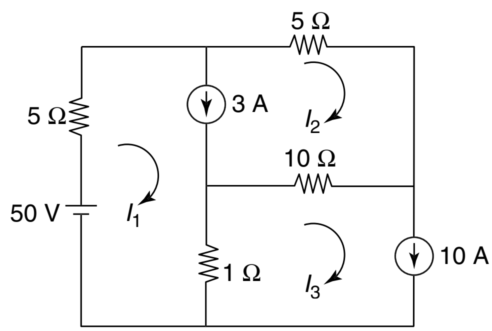

Fig. 2.83

Solution Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_1-I_2=3 \qquad \text{...(i)}$$

Applying KVL to the outer path of the supermesh,

$$50-5I_1-5I_2-10(I_2-I_3)-1(I_1-I_3)=0$$

$$-6I_1-15I_2+11I_3=-50 \qquad \text{...(ii)}$$

For Mesh 3,

$$I_3=10 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=9.76 \text{ A}$$

$$I_2=6.76 \text{ A}$$

$$I_3=10 \text{ A}$$

Power delivered by the voltage source = 50 I_1 = 50 × 9.76 = 488 W

$$I_{10\,\Omega}=I_3-I_2=10-6.76=3.24 \text{ A}$$

Example 2.51 *For the network shown in Fig. 2.84, find current through the 8 Ω resistor.*

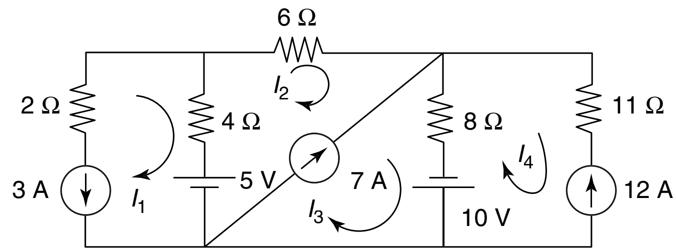

Fig. 2.84

Writing current equations for Meshes 1 and 4,

$$I_1 = -3 \quad \text{...(i)}$$

$$I_4 = -12 \quad \text{...(ii)}$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 7 \quad \text{...(iii)}$$

Applying KVL to the outer path of the supermesh,

$$5 - 4(I_2 - I_1) - 6I_2 - 8(I_3 - I_4) + 10 = 0$$

$$5 - 4(I_2 + 3) - 6I_2 - 8(I_3 + 12) + 10 = 0$$

$$-10I_2 - 8I_3 = 93 \quad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$I_2 = -8.28 \text{ A}$$

$$I_3 = -1.28 \text{ A}$$

$$I_{8\Omega} = I_3 - I_4 = -1.28 + 12 = 10.72 \text{ A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.52 *In the network of Fig. 2.85, find currents I_1 and I_2.*

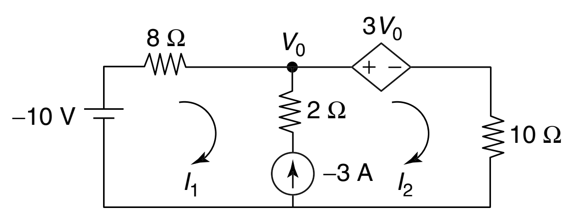

Fig. 2.85

Solution From Fig. 2.85,

$$-10 - 8I_1 - V_0 = 0$$

$$V_0 = -10 - 8I_1 \quad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equations for the supermesh,

$$I_2 - I_1 = -3 \quad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10 - 8I_1 - 3V_0 - 10I_2 = 0$$

$$-10 - 8I_1 - 3(-10 - 8I_1) - 10I_2 = 0$$

$$16I_1 - 10I_2 = -20 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -8.33 \text{ A}$$

$$I_2 = -11.33 \text{ A}$$

Example 2.53 *In the network of Fig. 2.86, find the current through the 3 Ω resistor.*

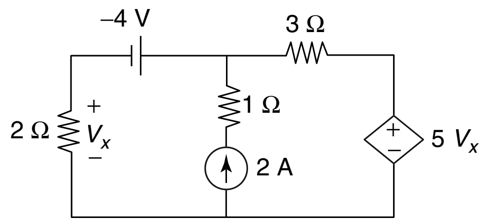

Fig. 2.86

Solution Assigning clockwise currents in two meshes as shown in Fig. 2.87.
From Fig. 2.87,

$$V_x = -2I_1 \qquad \ldots\text{(i)}$$

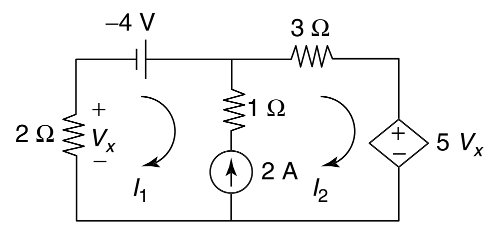

Fig. 2.87

Meshes 1 and 2 will form a supermesh.
Writing current equations for the supermesh,

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-2I_1 - 4 - 3I_2 - 5V_x = 0$$

$$-2I_1 - 4 - 3I_2 - 5(-2I_1) = 0$$

$$8I_1 - 3I_2 = 4 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$

$$I_2 = 4 \text{ A}$$

$$I_{3\,\Omega} = I_2 = 4 \text{ A}$$

Example 2.54 *Find the currents I_1 and I_2 at the network shown in Fig. 2.88.*

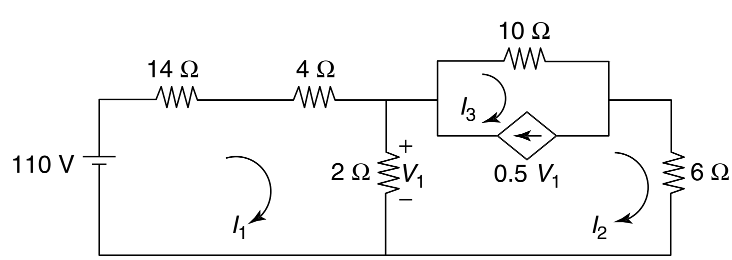

Fig. 2.88

Solution From Fig. 2.88,

$$V_1 = 2(I_1 - I_2)$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 0.5\ V_1 = 0.5 \times 2(I_1 - I_2) = I_1 - I_2$$
$$I_3 = I_1$$

Applying KVL to outer path of the supermesh,

$$-2(I_2 - I_1) - 10 I_3 - 6 I_2 = 0$$
$$-2 I_2 + 2 I_1 - 10 I_1 - 6 I_2 = 0$$
$$I_1 = -I_2$$

Applying KVL to Mesh 1,

$$110 - 14 I_1 - 4 I_1 - 2(I_1 - I_2) = 0$$
$$110 - 20 I_1 + 2 I_2 = 0$$
$$110 + 20 I_1 + 2 I_2 = 0$$
$$I_2 = -5 \text{ A}$$
$$I_1 = -I_2 = 5 \text{ A}$$

Example 2.55 *For the network of Fig. 2.89, find current through the 8 Ω resistor.*

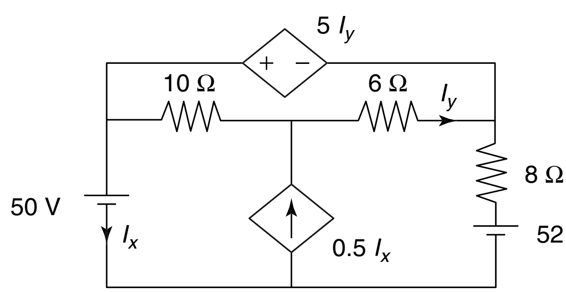

Fig. 2.89

Solution Assigning clockwise currents to the three meshes as shown in Fig. 2.90.
From Fig. 2.90,

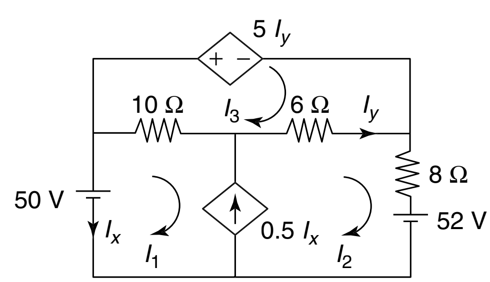

Fig. 2.90

$$I_x = -I_1 \qquad \ldots\text{(i)}$$
$$I_y = I_2 - I_3 \qquad \ldots\text{(ii)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 0.5\ I_x = 0.5\,(-I_1)$$
$$-0.5\ I_1 + I_2 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to the outer path of the supermesh,

$$50 - 10(I_1 - I_3) - 6(I_2 - I_3) - 8 I_2 - 52 = 0$$
$$-10 I_1 - 14 I_2 + 16 I_3 = 2 \qquad \ldots\text{(iv)}$$

Applying KVL to Mesh 3,

$$-5\,I_y - 6\,(I_3 - I_2) - 10\,(I_3 - I_1) = 0$$

$$-5\,(I_2 - I_3) - 6\,(I_3 - I_2) - 10\,(I_3 - I_1) = 0$$

$$10\,I_1 + I_2 - 11\,I_3 = 0 \qquad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = -1.56 \text{ A}$$
$$I_2 = -0.58 \text{ A}$$
$$I_3 = -1.11 \text{ A}$$
$$I_{8\,\Omega} = I_2 = -0.58 \text{ A}$$

Example 2.56 *For the network shown in Fig. 2.91, find the current through the 10 Ω resistor.*

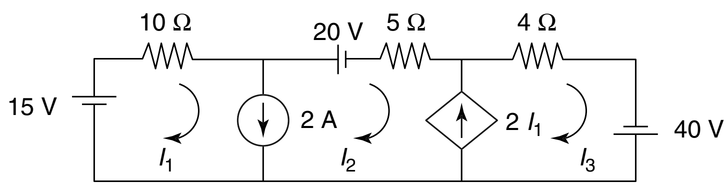

Fig. 2.91

Solution Meshes 1, 2 and 3 will form a supermesh.
Writing current equations for the supermesh,

$$I_1 - I_2 = 2 \qquad \text{...(i)}$$

$$I_3 - I_2 = 2\,I_1 \qquad \text{...(ii)}$$

and

$$2\,I_1 + I_2 - I_3 = 0$$

Applying KVL to the outer path of the supermesh,

$$15 - 10\,I_1 - 20 - 5\,I_2 - 4I_3 + 40 = 0$$

$$10\,I_1 + 5\,I_2 + 4\,I_3 = 35 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 1.96 \text{ A}$$
$$I_2 = -0.04 \text{ A}$$
$$I_3 = 3.89 \text{ A}$$
$$I_{10\,\Omega} = I_1 = 1.96 \text{ A}$$

Example 2.57 *In the network shown in Fig. 2.92, find the power delivered by the 4 V source and voltage across the 2 Ω resistor.*

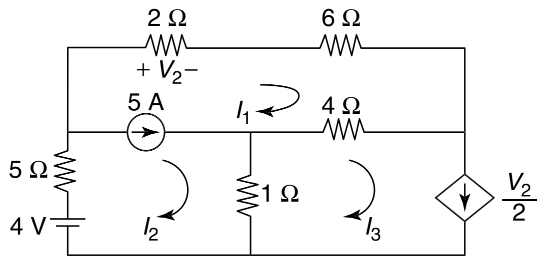

Fig. 2.92

Solution From Fig. 2.92,

$$V_2 = 2\,I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 5 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$4 - 5\,I_2 - 2\,I_1 - 6\,I_1 - 4(I_1 - I_3) - 1(I_2 - I_3) = 0$$

$$-12\,I_1 - 6\,I_2 + 5\,I_3 = -4 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = \frac{V_2}{2} = \frac{2\,I_1}{2} = I_1$$

$$I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -2\text{ A}$$

$$I_2 = 3\text{ A}$$

$$I_3 = -2\text{ A}$$

Power delivered by the 4 V source $= 4\,I_2 = 4(3) = 12$ W

$$V_{2\,\Omega} = 2\,I_1 = 2(-2) = -4\text{ V}$$

Example 2.58 *Find currents I_1, I_2, I_3, I_4 of the network shown in Fig. 2.93.*

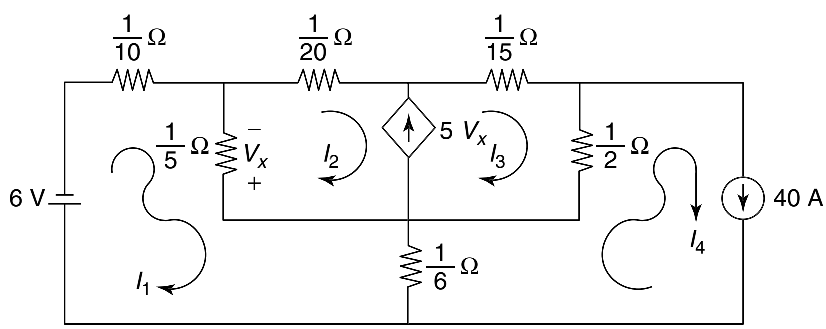

Fig. 2.93

From Fig. 2.93,

$$V_x = \frac{1}{5}(I_2 - I_1) \qquad \ldots\text{(i)}$$

For Mesh 4,

$$I_4 = 40 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}(I_1 - I_2) - \frac{1}{6}(I_1 - I_4) = 0$$

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}I_1 + \frac{1}{5}I_2 - \frac{1}{6}I_1 + \frac{1}{6}(40) = 0$$

$$-\frac{7}{15}I_1 + \frac{1}{5}I_2 = -\frac{2}{3} \qquad \ldots\text{(iii)}$$

Mesh 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 5\,V_x = 5\left[\frac{1}{5}(I_2 - I_1)\right] = I_2 - I_1$$

$$I_1 - 2I_2 + I_3 = 0 \qquad \ldots\text{(iv)}$$

Applying KVL to the outer path of the supermesh,

$$-\frac{1}{5}(I_2 - I_1) - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}(I_3 - I_4) = 0$$

$$-\frac{1}{5}I_2 + \frac{1}{5}I_1 - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}I_3 + \frac{1}{2}(40) = 0$$

$$\frac{1}{5}I_1 - \frac{1}{4}I_2 - \frac{17}{30}I_3 = -20 \qquad \ldots\text{(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = 10 \text{ A}$$
$$I_2 = 20 \text{ A}$$
$$I_3 = 30 \text{ A}$$
$$I_4 = 40 \text{ A}$$

2.5 NODE ANALYSIS

Node analysis is based on Kirchhoff's current law which states that the algebraic sum of currents meeting at a point is zero. Every junction where two or more branches meet is regarded as a node. One of the nodes in the network is taken as *reference node* or *datum node*. If there are *n* nodes in any network, the number of simultaneous equations to be solved will be $(n - 1)$.

Steps to be followed in Node Analysis

1. Assuming that a network has *n* nodes, assign a reference node and the reference directions, and assign a current and a voltage name for each branch and node respectively.
2. Apply KCL at each node except for the reference node and apply Ohm's law to the branch currents.
3. Solve the simultaneous equations for the unknown node voltages.
4. Using these voltages, find any branch currents required.

Example 2.59 *Calculate the current through 2 Ω resistor for the network shown in Fig. 2.94.*

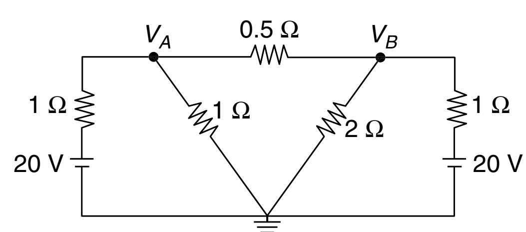

Fig. 2.94

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node A,

$$\frac{V_A - 20}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{0.5}\right) V_A - \frac{1}{0.5} V_B = \frac{20}{1}$$

$$4\,V_A - 2\,V_B = 20 \qquad \text{...(i)}$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$-\frac{1}{0.5} V_A \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{1}\right) V_B = \frac{20}{1}$$

$$-2\,V_A + 3.5\,V_B = 20 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$V_A = 11 \text{ V}$$

$$V_B = 12 \text{ V}$$

Current through the 2 Ω resistor $= \dfrac{V_B}{2} = \dfrac{12}{2} = 6$ A

Example 2.60 *Find the voltage at nodes 1 and 2 for the network shown in Fig. 2.95.*

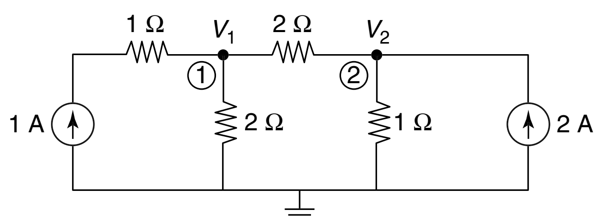

Fig. 2.95

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{2} + \frac{1}{2}\right) V_1 - \frac{1}{2} V_2 = 1$$

$$V_1 - 0.5\,V_2 = 1 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$2 = \frac{V_2}{1} + \frac{V_2 - V_1}{2}$$

$$-\frac{1}{2} V_1 + \left(1 + \frac{1}{2}\right) V_2 = 2$$

$$-0.5 V_1 + 1.5\,V_2 = 4 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$V_1 = 2\text{ V}$$
$$V_2 = 2\text{ V}$$

Example 2.61 *Find the current in the 100 Ω resistor for the network shown in Fig. 2.96.*

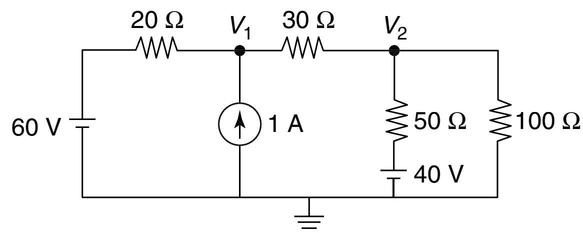

Fig. 2.96

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1-60}{20}+\frac{V_1-V_2}{30}=1$$
$$\left(\frac{1}{20}+\frac{1}{30}\right)V_1-\frac{1}{30}V_2=\frac{60}{20}+1$$
$$0.083\,V_1-0.033\,V_2=4 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{30}+\frac{V_2-40}{50}+\frac{V_2}{100}=0$$
$$-\frac{1}{30}V_1+\left(\frac{1}{30}+\frac{1}{50}+\frac{1}{100}\right)V_2=\frac{40}{50}$$
$$-0.033\,V_1+0.063\,V_2=0.8 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$V_1 = 67.25\text{ V}$$
$$V_2 = 48\text{ V}$$

$$\text{Current through the 100 }\Omega\text{ resistor} = \frac{V_2}{100}=\frac{48}{100}=0.48\text{ A}$$

Example 2.62 *Find V_A and V_B for the network shown in Fig. 2.97.*

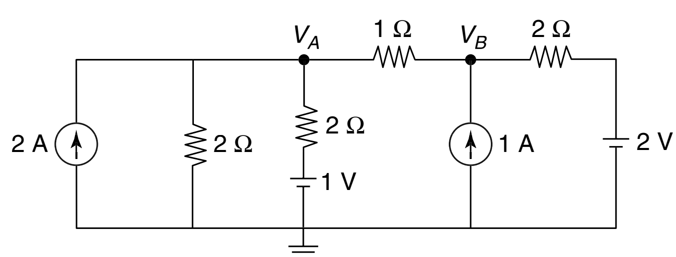

Fig. 2.97

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node A,

$$2 = \frac{V_A}{2} + \frac{V_A - 1}{2} + \frac{V_A - V_B}{1}$$

$$\left(\frac{1}{2} + \frac{1}{2} + 1\right)V_A - V_B = 2 + \frac{1}{2}$$

$$2V_A - V_B = 2.5 \qquad \ldots\text{(i)}$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{1} + \frac{V_B - 2}{2} = 1 \qquad \ldots\text{(ii)}$$

$$-V_A + \left(1 + \frac{1}{2}\right)V_B = 1 + \frac{2}{2}$$

$$-V_A + 1.5\,V_B = 2$$

Solving Eqs (i) and (ii),

$$V_A = 2.875 \text{ V}$$

$$V_B = 3.25 \text{ V}$$

Example 2.63 *Find currents I_1, I_2 and I_3 for the network shown in Fig. 2.98.*

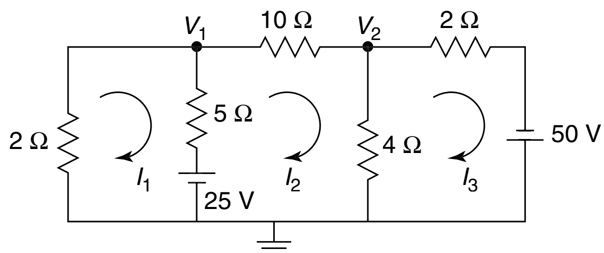

Fig. 2.98

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1}{2} + \frac{V_1 - 25}{5} + \frac{V_1 - V_2}{10} = 0$$

$$\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)V_1 - \frac{1}{10}V_2 = \frac{25}{5}$$

$$0.8\,V_1 - 0.1\,V_2 = 5 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{4} + \frac{V_2 - (-50)}{2} = 0$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + \frac{1}{4} + \frac{1}{2}\right)V_2 = -\frac{50}{2}$$

$$-0.1\,V_1 + 0.85\,V_2 = -25 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$V_1 = 2.61\text{V}$$
$$V_2 = -29.1\text{ V}$$
$$I_1 = -\frac{V_1}{2} = -\frac{2.61}{2} = -1.31\text{ A}$$
$$I_2 = \frac{V_1 - V_2}{10} = \frac{2.61-(-2.91)}{10} = 3.17\text{A}$$
$$I_3 = \frac{V_2 + 50}{2} = \frac{-29.1+50}{2} = 10.45\text{ A}$$

Example 2.64 *Find currents I_1, I_2 and I_3 and voltages V_a and V_b for the network shown in Fig. 2.99.*

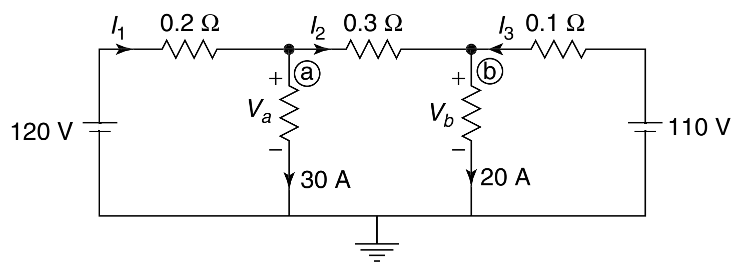

Fig. 2.99

Solution Applying KCL at Node a,

$$I_1 = 30 + I_2$$
$$\frac{120 - V_a}{0.2} = 30 + \frac{V_a - V_b}{0.3}$$
$$36 - 0.3\,V_a = 1.8 + 0.2V_a - 0.2\,V_b$$
$$0.5\,V_a - 0.2\,V_b = 34.2 \qquad \ldots\text{(i)}$$

Applying KCL at Node b,

$$I_2 + I_3 = 20$$
$$\frac{V_a - V_b}{0.3} + \frac{110 - V_b}{0.1} = 20$$
$$\frac{0.1\,V_a - 0.1\,V_b + 33 - 0.3\,V_b}{0.03} = 20$$
$$0.1\,V_a - 0.4\,V_b = -32.4 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$V_a = 112\text{ V}$$
$$V_b = 109\text{ V}$$
$$I_1 = \frac{120 - V_a}{0.2} = \frac{120-112}{0.2} = 40\text{ A}$$
$$I_2 = \frac{V_a - V_b}{0.3} = \frac{112-109}{0.3} = 10\text{ A}$$
$$I_3 = \frac{110 - V_b}{0.1} = \frac{110-109}{0.1} = 10\text{ A}$$

Example 2.65 *Calculate the current through the 5 Ω resistor for the network shown in Fig. 2.100.*

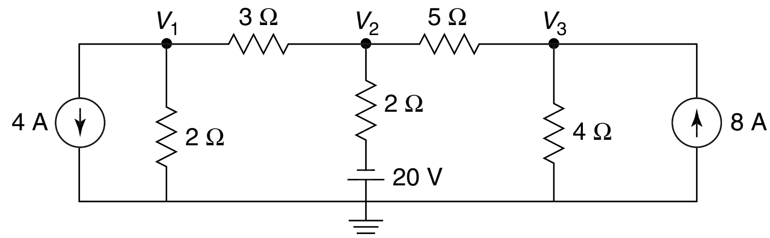

Fig. 2.100

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$4+\frac{V_1}{2}+\frac{V_1-V_2}{3}=0$$

$$\left(\frac{1}{2}+\frac{1}{3}\right)V_1-\frac{1}{3}V_2=4$$

$$0.83\,V_1-0.33\,V_2=-4 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{3}+\frac{V_2-(-20)}{2}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{3}V_1+\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=-\frac{20}{2}$$

$$-0.33\,V_1+1.03\,V_2-0.2\,V_3=-10 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{4}=8$$

$$-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{4}\right)V_3=8$$

$$-0.2\,V_2+0.45\,V_3=8 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=-8.76\text{ V}$$

$$V_2=-9.92\text{ V}$$

$$V_3=13.37\text{ V}$$

$$\text{Current through the } 5\ \Omega \text{ resistor} = \frac{V_3-V_2}{5}=\frac{13.37-(-9.92)}{5}=4.66\text{ A}$$

Example 2.66 *Find the voltage across the 5 Ω resistor for the network shown in Fig. 2.101.*

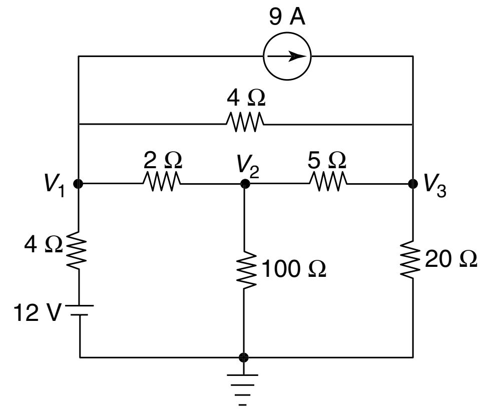

Fig. 2.101

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1-12}{4}+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{4}+9=0$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=-9+\frac{12}{4}$$

$$V_1-0.5\,V_2-0.25\,V_3=-6 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5\,V_1+0.71\,V_2-0.2\,V_3=0 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-V_1}{4}=9$$

$$-\frac{1}{4}V_1-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3=9$$

$$-0.25\,V_1-0.2\,V_2+0.5\,V_3=9 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=6.35\text{ V}$$

$$V_2=11.76\text{ V}$$

$$V_3=25.88\text{ V}$$

Voltages across the 5 Ω resistor $= V_3-V_2=25.88-11.76=14.12\text{ V}$

Example 2.67 *Determine the current through the 5 Ω resistor for the network shown in Fig. 2.102.*

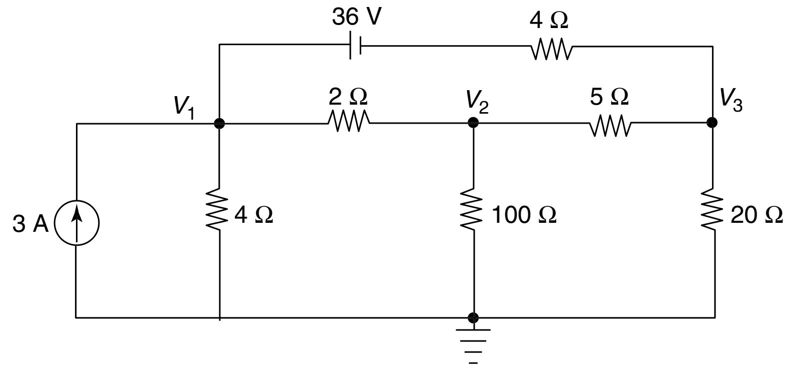

Fig. 2.102

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1}{4}+\frac{V_1-V_2}{2}+\frac{V_1-36-V_3}{4}=3$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=3+\frac{36}{4}$$

$$V_1-0.5\,V_2-0.25\,V_3=12 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5\,V_1+0.71\,V_2-0.2\,V_3=0 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-(-36)-V_1}{4}=0$$

$$-\frac{1}{4}V_1-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3=-9$$

$$-0.25\,V_1-0.2\,V_2+0.5\,V_3=-9 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=13.41\text{ V}$$
$$V_2=7.06\text{ V}$$
$$V_3=-8.47\text{ V}$$

$$\text{Current through the 5 Ω resistor}=\frac{V_2-V_3}{5}=\frac{7.06-(-8.47)}{5}=3.11\text{ A}$$

Example 2.68 *Find the voltage drop across the 5 Ω resistor in the network shown in Fig. 2.103.*

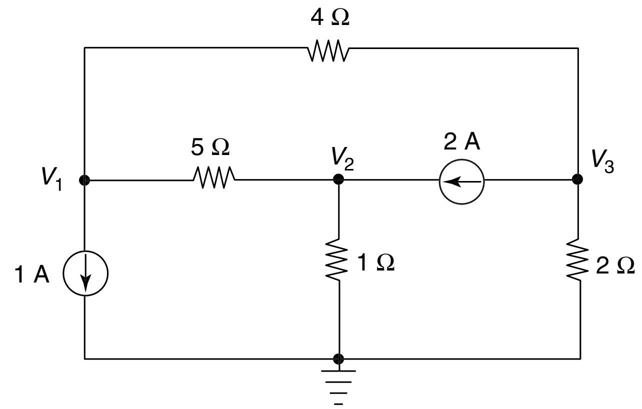

Fig 2.103

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$1+\frac{V_1-V_2}{5}+\frac{V_1-V_3}{4}=0$$

$$\left(\frac{1}{5}+\frac{1}{4}\right)V_1-\frac{1}{5}V_2-\frac{1}{4}V_3=-1$$

$$0.45\,V_1-0.2\,V_2-0.25\,V_3=-1 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{5}+\frac{V_2}{1}=2$$

$$-\frac{1}{5}V_1+\left(\frac{1}{5}+1\right)V_2=2$$

$$-0.2\,V_1+1.2\,V_2=2 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3}{2}+\frac{V_3-V_1}{4}+2=0$$

$$-\frac{1}{4}V_1+\left(\frac{1}{2}+\frac{1}{4}\right)V_3=-2$$

$$-0.25\,V_1+0.75\,V_3=-2 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=-4\text{ V}$$

$$V_2=1\text{ V}$$

$$V_3=-4\text{ V}$$

$$V_{5\,\Omega}=V_2-V_1=1-(-4)=5\text{ V}$$

Example 2.69 *Find the power dissipated in the 6 Ω resistor for the network shown in Fig. 2.104.*

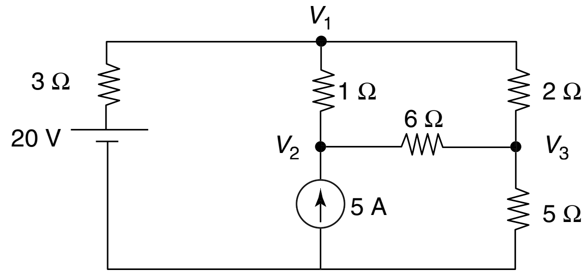

Fig. 2.104

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1-20}{3}+\frac{V_1-V_2}{1}+\frac{V_1-V_3}{2}=0$$

$$\left(\frac{1}{3}+1+\frac{1}{2}\right)V_1-V_2-\frac{1}{2}V_3=\frac{20}{3}$$

$$1.83\,V_1-V_2-0.5\,V_3=6.67 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{1}+\frac{V_2-V_3}{6}=5$$

$$-V_1+\left(1+\frac{1}{6}\right)V_2-\frac{1}{6}V_3=5$$

$$-V_1+1.17\,V_2-0.17\,V_3=5 \qquad \text{...(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_1}{2}+\frac{V_3}{5}+\frac{V_3-V_2}{6}=0$$

$$-\frac{1}{2}V_1-\frac{1}{6}V_2+\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{6}\right)V_3=0$$

$$-0.5\,V_1-0.17\,V_2+0.87\,V_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=23.82\text{ V}$$

$$V_2=27.4\text{ V}$$

$$V_3=19.04\text{ V}$$

$$I_{6\,\Omega}=\frac{V_2-V_3}{6}=\frac{27.4-19.04}{6}=1.39\text{ A}$$

Power dissipated in the 6 Ω resistor $=(1.39)^2\times 6=11.59$ W

Example 2.70 *Find the voltage V in the network shown in Fig. 2.105 which makes the current in the 10 Ω resistor zero.*

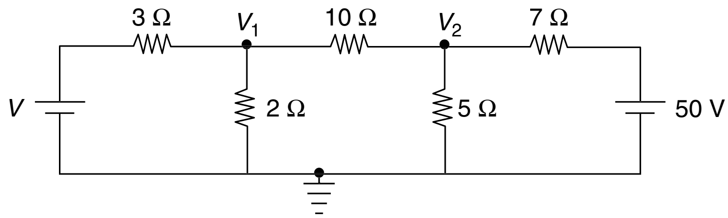

Fig. 2.105

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1 - V}{3} + \frac{V_1}{2} + \frac{V_1 - V_2}{10} = 0$$

$$\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{10}\right)V_1 - \frac{1}{10}V_2 - \frac{1}{3}V = 0$$

$$0.93\,V_1 - 0.1\,V_2 - 0.33\,V = 0 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + \frac{V_2 - 50}{7} = 0$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{7}\right)V_2 = \frac{50}{7}$$

$$-0.1\,V_1 + 0.44\,V_2 = 7.14 \qquad \ldots\text{(ii)}$$

$$I_{10\,\Omega} = \frac{V_1 - V_2}{10} = 0$$

$$V_1 - V_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V = 52.82 \text{ V}$$

Example 2.71 *Find V_1 and V_2 for the network shown in Fig. 2.106.*

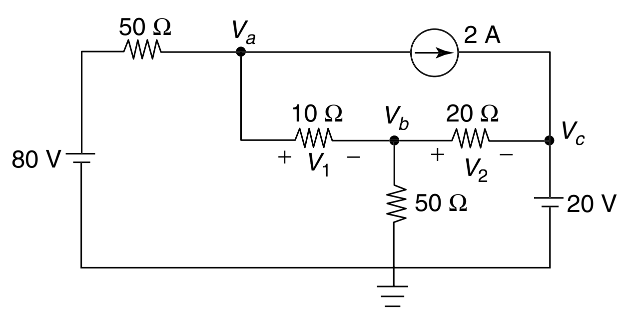

Fig. 2.106

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node a,

$$\frac{V_a - 80}{50} + \frac{V_a - V_b}{10} + 2 = 0$$

$$\left(\frac{1}{50} + \frac{1}{10}\right) V_a - \frac{1}{10} V_b = \frac{80}{50} - 2$$

$$0.12\, V_a - 0.1\, V_b = -0.4 \qquad \ldots\text{(i)}$$

Applying KCL at Node b,

$$\frac{V_b - V_a}{10} + \frac{V_b}{50} + \frac{V_b - V_c}{20} = 0$$

$$-\frac{1}{10} V_a + \left(\frac{1}{10} + \frac{1}{50} + \frac{1}{20}\right) V_b - \frac{1}{20} V_c = 0$$

$$-0.1\, V_a + 0.17\, V_b - 0.05\, V_c = 0 \qquad \ldots\text{(ii)}$$

Node c is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node c.

$$V_c = 20 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_a = 3.08 \text{ V}$$

$$V_b = 7.69 \text{ V}$$

$$V_1 = V_a - V_b = 3.08 - 7.69 = -4.61 \text{ V}$$

$$V_2 = V_b - V_c = 7.69 - 20 = -12.31 \text{ V}$$

Example 2.72 *Find the voltage across the 100 Ω resistor for the network shown in Fig. 2.107.*

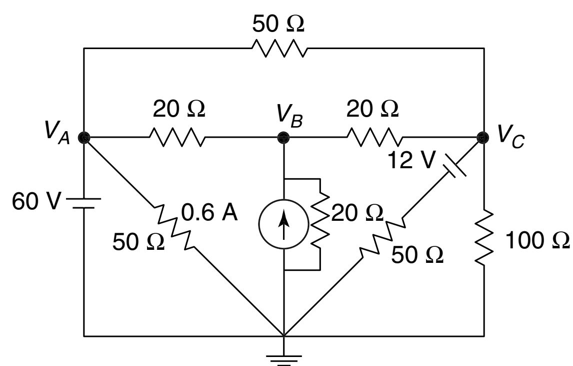

Fig. 2.107

Solution

Node A is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node A.

$$V_A = 60 \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node B,

$$\frac{V_B - V_A}{20} + \frac{V_B - V_C}{20} + \frac{V_B}{20} = 0.6$$

$$-\frac{1}{20} V_A + \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{20}\right) V_B - \frac{1}{20} V_C = 0.6$$

$$-0.05\, V_A + 0.15\, V_B - 0.05\, V_C = 0.6 \qquad \ldots\text{(ii)}$$

Applying KCL at Node C,

$$\frac{V_C - V_A}{50} + \frac{V_C - V_B}{20} + \frac{V_C - 12}{50} + \frac{V_C}{100} = 0$$

$$-\frac{1}{50}V_A - \frac{1}{20}V_B + \left(\frac{1}{50} + \frac{1}{20} + \frac{1}{50} + \frac{1}{100}\right)V_C = \frac{12}{50}$$

$$-0.02\,V_A - 0.05\,V_B + 0.1\,V_C = 0.24 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_C = 31.68 \text{ V}$$

Voltages across the 100 Ω resistor = 31.68 V

EXAMPLES WITH DEPENDENT SOURCES

Example 2.73 *Find the voltage across the 5 Ω resistor in the network shown in Fig. 2.108.*

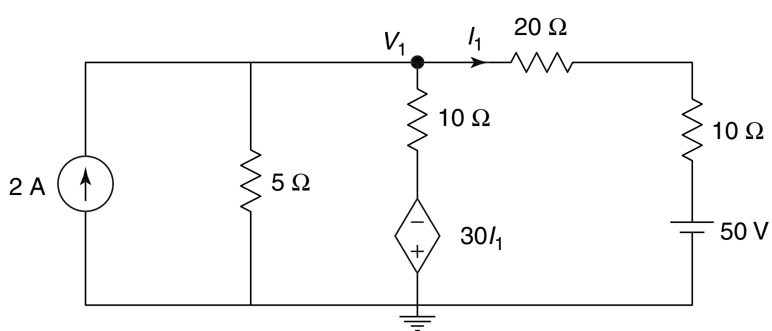

Fig. 2.108

Solution From Fig. 2.108,

$$I_1 = \frac{V_1 - 50}{20 + 10} = \frac{V_1 - 50}{30} \qquad \text{...(i)}$$

Assume that the currents are moving away from the node.
Applying KCL at Node 1,

$$2 = \frac{V_1}{5} + \frac{V_1 + 30\,I_1}{10} + \frac{V_1 - 50}{30}$$

$$2 = \frac{V_1}{5} + \frac{V_1 + 30\left(\dfrac{V_1 - 50}{30}\right)}{10} + \frac{V_1 - 50}{30}$$

$$2 = \frac{V_1}{5} + \frac{2V_1 - 50}{10} + \frac{V_1 - 50}{30} \qquad \text{...(ii)}$$

Solving Eq. (ii),

$$V_1 = 20 \text{ V}$$

Voltage across the 5 Ω resistor = 20 V

Example 2.74 *For the network shown in Fig. 2.109, find the voltage V_x.*

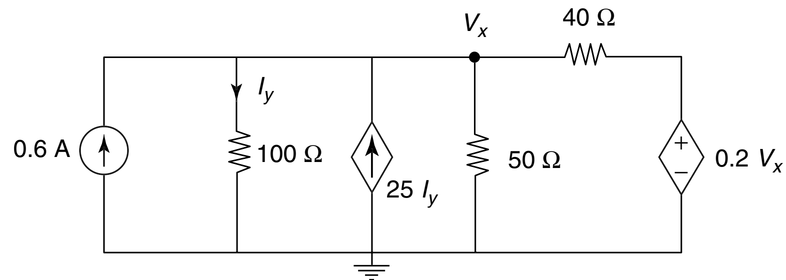

Fig. 2.109

Solution From Fig. 2.109,

$$I_y = \frac{V_x}{100} \qquad \text{...(i)}$$

Assume that the currents are moving away from the node.
Applying KCL at Node x,

$$25\,I_y + 0.6 = \frac{V_x}{100} + \frac{V_x}{50} + \frac{V_x - 0.2\,V_x}{40}$$

$$25\left(\frac{V_x}{100}\right) + 0.6 = \frac{V_x}{100} + \frac{V_x}{50} + \frac{0.8\,V_x}{40}$$

$$\left(\frac{1}{4} - \frac{1}{100} - \frac{1}{50} - \frac{0.8}{40}\right)V_x = -0.6$$

$$0.2\,V_x = -0.6$$

$$V_x = -3\text{ V}$$

Example 2.75 *For the network shown in Fig. 2.110, find voltages V_1 and V_2.*

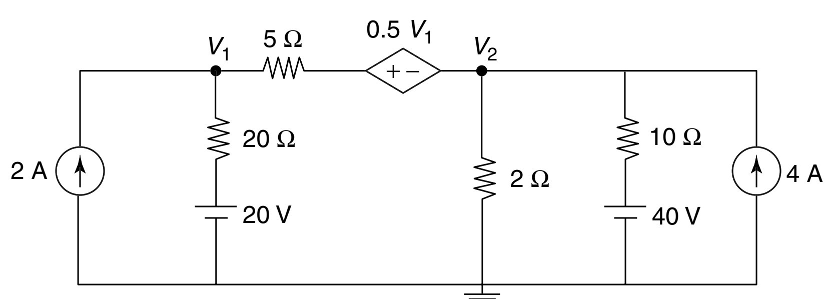

Fig. 2.110

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$2 = \frac{V_1 - 20}{20} + \frac{V_1 - 0.5\,V_1 - V_2}{5}$$

$$\left(\frac{1}{20} + \frac{1}{5} - \frac{0.5}{5}\right)V_1 - \frac{1}{5}V_2 = 3$$

$$0.15\,V_1 - 0.2\,V_2 = 3 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2+0.5\,V_1-V_1}{5}+\frac{V_2}{2}+\frac{V_2-40}{10}=4$$
$$\left(\frac{0.5}{5}-\frac{1}{5}\right)V_1+\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{10}\right)V_2=4+\frac{40}{10}$$
$$-0.1\,V_1+0.8\,V_2=8 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$V_1=40\text{ V}$$
$$V_2=15\text{ V}$$

Example 2.76 *Determine the voltages V_1 and V_2 in the network of Fig. 2.111.*

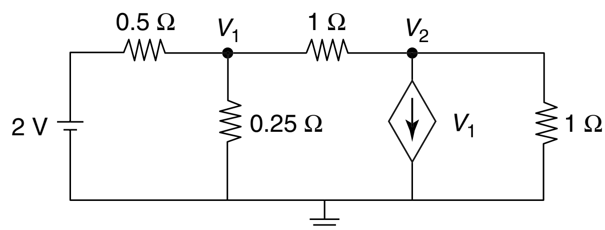

Fig. 2.111

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1-2}{0.5}+\frac{V_1}{0.25}+\frac{V_1-V_2}{1}=0$$
$$\left(\frac{1}{0.5}+\frac{1}{0.25}+1\right)V_1-V_2=\frac{2}{0.5}$$
$$7\,V_7-V_2=4 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{1}+\frac{V_2}{1}+V_1=0$$
$$2\,V_2=0$$
$$V_2=0 \qquad \ldots\text{(ii)}$$

From Eq. (i),

$$V_1=\frac{4}{7}\text{ V}$$

Example 2.77 *In the network of Fig. 2.112, find the node voltages V_1, V_2 and V_3.*

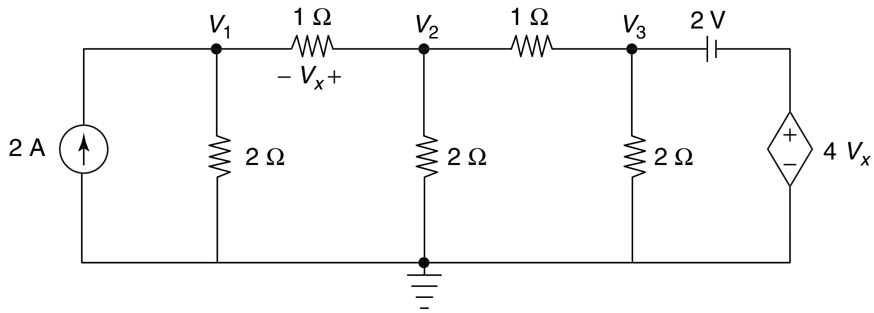

Fig. 2.112

Solution From Fig. 2.112,

$$V_x = V_2 - V_1 \qquad \text{...(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$2 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$

$$\left(\frac{1}{2} + 1\right) V_1 - V_2 = 2$$

$$1.5\, V_1 - V_2 = 2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$$

$$-V_1 + \left(1 + \frac{1}{2} + 1\right) V_2 - V_3 = 0$$

$$-V_1 + 2.5\, V_2 - V_3 = 0 \qquad \text{...(iii)}$$

At Node 3,

$$V_3 - 4\, V_x = 2$$

$$V_3 - 4(V_2 - V_1) = 2$$

$$4\, V_1 - 4\, V_2 + V_3 = 2 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$V_1 = -1.33 \text{ V}$$

$$V_2 = -4 \text{ V}$$

$$V_3 = -8.67 \text{ V}$$

Example 2.78 *For the network shown in Fig. 2.113, find the node voltages V_1 and V_2.*

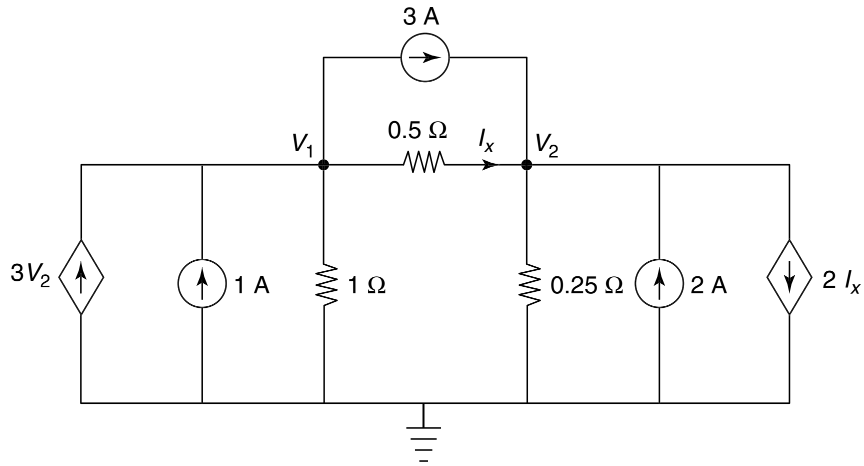

Fig. 2.113

Solution From Fig. 2.113,

$$I_x = \frac{V_1 - V_2}{0.5} \qquad \text{...(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$3\,V_2 + 1 = \frac{V_1}{1} + \frac{V_1 - V_2}{0.5} + 3$$

$$\left(1 + \frac{1}{0.5}\right)V_1 - \left(3 + \frac{1}{0.5}\right)V_2 = -2$$

$$3\,V_1 - 5\,V_2 = -2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$3 + 2 = \frac{V_2 - V_1}{0.5} + \frac{V_2}{0.25} + 2\,I_x$$

$$5 = \frac{V_2 - V_1}{0.5} + \frac{V_2}{0.25} + 2\left(\frac{V_1 - V_2}{0.5}\right)$$

$$\left(-\frac{1}{0.5} + \frac{2}{0.5}\right)V_1 + \left(\frac{1}{0.5} + \frac{1}{0.25} - \frac{2}{0.5}\right)V_2 = 5$$

$$2\,V_1 + 2\,V_2 = 5 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1 = 1.31\ \text{V}$$

$$V_2 = 1.19\ \text{V}$$

Example 2.79 *Find voltages V_1 and V_2 in the network shown in Fig. 2.114.*

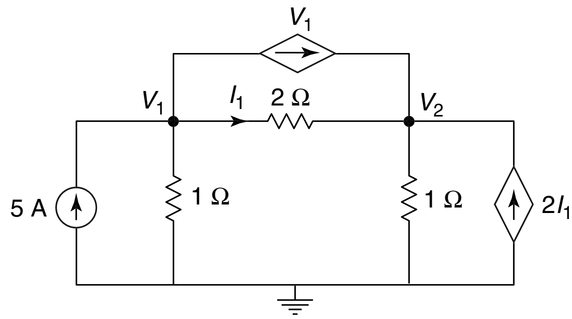

Fig. 2.114

Solution From Fig. 2.114,

$$I_1 = \frac{V_1 - V_2}{2} \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$5 = \frac{V_1}{1} + \frac{V_1 - V_2}{2} + V_1$$

$$\left(1 + \frac{1}{2} + 1\right) V_1 - \frac{1}{2} V_2 = 5$$

$$2.5\,V_1 - 0.5\,V_2 = 5 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} = 2I_1 + V_1$$

$$V_2 - V_1 + V_2 = 2\left(\frac{V_1 - V_2}{2}\right) + V_1$$

$$3\,V_1 = 3\,V_2$$

$$V_1 = V_2 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1 = 2.5 \text{ V}$$

$$V_2 = 2.5 \text{ V}$$

Example 2.80 *Find the power supplied by the 10 V source in the network shown in Fig. 2.115.*

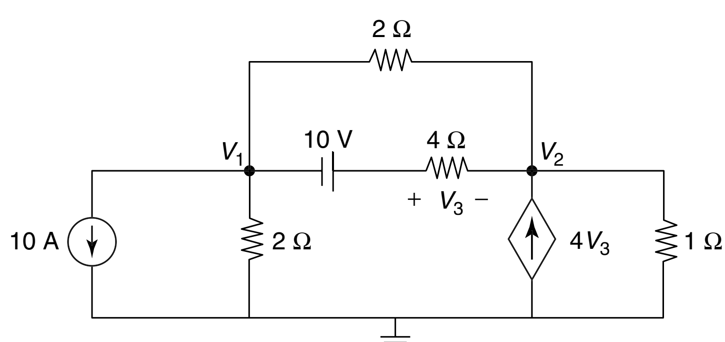

Fig. 2.115

Solution Assume that the currents are moving away from the nodes.
From Fig. 2.115,

$$V_3 = V_1 + 10 - V_2 \qquad \text{...(i)}$$

Applying KCL at Node 1,

$$10 + \frac{V_1}{2} + \frac{V_1 + 10 - V_2}{4} + \frac{V_1 - V_2}{2} = 0$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{2}\right)V_1 - \left(\frac{1}{4} + \frac{1}{2}\right)V_2 = -10 - \frac{10}{4}$$

$$1.25\,V_1 - 0.75\,V_2 = -12.5 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - 10 - V_1}{4} + \frac{V_2 - V_1}{2} + \frac{V_2}{1} = 4\,V_3$$

$$\frac{V_2 - 10 - V_1}{4} + \frac{V_2 - V_1}{2} + \frac{V_2}{1} = 4(V_1 + 10 - V_2)$$

$$\left(-\frac{1}{4} - \frac{1}{2} - 4\right)V_1 + \left(\frac{1}{4} + \frac{1}{2} + 1 + 4\right)V_2 = \frac{10}{4} + 40$$

$$-4.75\,V_1 + 5.75\,V_2 = 42.5 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1 = -11.03 \text{ V}$$

$$V_2 = -1.72 \text{ V}$$

$$I_{10\,\text{V}} = \frac{V_1 + 10 - V_2}{4} = \frac{-11.03 + 10 - (-1.72)}{4} = 0.173 \text{ A}$$

Power supplied by the 10 V source = $10 \times 0.173 = 1.73$ W

Example 2.81 *For the network shown in Fig. 2.116, find voltages V_1 and V_2.*

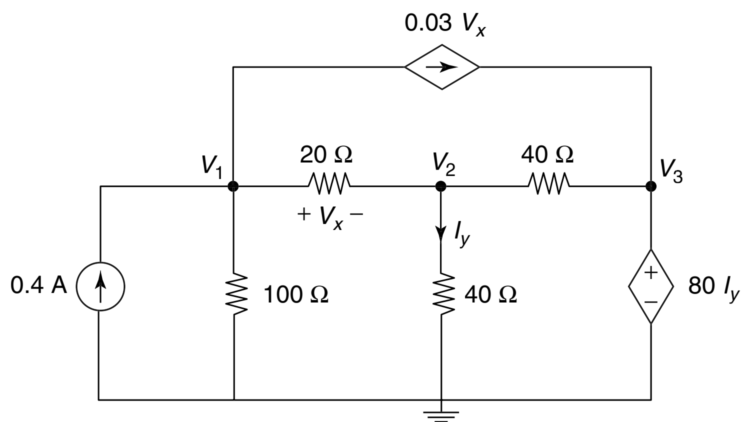

Fig. 2.116

Solution From Fig. 2.116,

$$V_x = V_1 - V_2 \qquad \text{...(i)}$$

$$I_y = \frac{V_2}{40} \qquad \text{...(ii)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$0.4 = \frac{V_1}{100} + \frac{V_1 - V_2}{20} + 0.03\, V_x$$

$$0.4 = \frac{V_1}{100} + \frac{V_1 - V_2}{20} + 0.03(V_1 - V_2)$$

$$\left(\frac{1}{100} + \frac{1}{20} + 0.03\right) V_1 - \left(\frac{1}{20} + 0.03\right) V_2 = 0.4$$

$$0.09\, V_1 - 0.08\, V_2 = 0.4 \qquad \ldots\text{(iii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{20} + \frac{V_2}{40} + \frac{V_2 - V_3}{40} = 0$$

$$-\frac{1}{20} V_1 + \left(\frac{1}{20} + \frac{1}{40} + \frac{1}{40}\right) V_2 - \frac{1}{40} V_3 = 0$$

$$-0.05\, V_1 + 0.1\, V_2 - 0.025\, V_3 = 0 \qquad \ldots\text{(iv)}$$

For Node 3,

$$V_3 = 80\, I_y = 80\left(\frac{V_2}{40}\right) = 2\, V_2$$

$$2\, V_2 - V_3 = 0 \qquad \ldots\text{(v)}$$

Solving Eqs (iii), (iv) and (v),

$$V_1 = 40 \text{ V}$$
$$V_2 = 40 \text{ V}$$
$$V_3 = 80 \text{ V}$$

Example 2.82 *Find voltages V_a, V_b and V_c in the network shown in Fig. 2.117.*

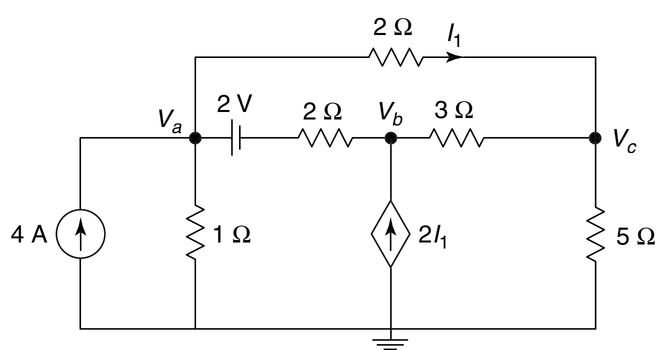

Fig. 2.117

Solution From Fig. 2.117,

$$I_1 = \frac{V_a - V_c}{2}$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node a,

$$4 = \frac{V_a}{1} + \frac{V_a - V_c}{2} + \frac{V_a - 2 - V_b}{2}$$

$$\left(1 + \frac{1}{2} + \frac{1}{2}\right)V_a - \frac{1}{2}V_b - \frac{1}{2}V_c = 5$$

$$2\,V_a - 0.5\,V_b - 0.5\,V_c = 5 \qquad \ldots\text{(i)}$$

Applying KCL at Node b,

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = 2I_1$$

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = 2\left(\frac{V_a - V_c}{2}\right)$$

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = V_a - V_c$$

$$\left(-\frac{1}{2} - 1\right)V_a + \left(\frac{1}{2} + \frac{1}{3}\right)V_b + \left(1 - \frac{1}{3}\right)V_c = -1$$

$$-1.5\,V_a + 0.83\,V_b + 0.67\,V_c = -1 \qquad \ldots\text{(ii)}$$

Applying KCL at Node c,

$$\frac{V_c - V_b}{3} + \frac{V_c}{5} = I_1$$

$$\frac{V_c - V_b}{3} + \frac{V_c}{5} = \frac{V_a - V_c}{2}$$

$$-\frac{1}{2}V_a - \frac{1}{3}V_b + \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right)V_c = 0$$

$$-0.5\,V_a - 0.33\,V_b + 1.033\,V_c = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_a = 4.303\text{ V}$$

$$V_b = 3.88\text{ V}$$

$$V_c = 3.33\text{ V}$$

2.6 SUPERNODE ANALYSIS

Nodes that are connected to each other by voltage sources, but not to the reference node by a path of voltage sources, form a *supernode*. A supernode requires one node voltage equation, that is, a KCL equation. The remaining node voltage equations are KVL equations.

Example 2.83 *Determine the current in the 5 Ω resistor for the network shown in Fig. 2.118.*

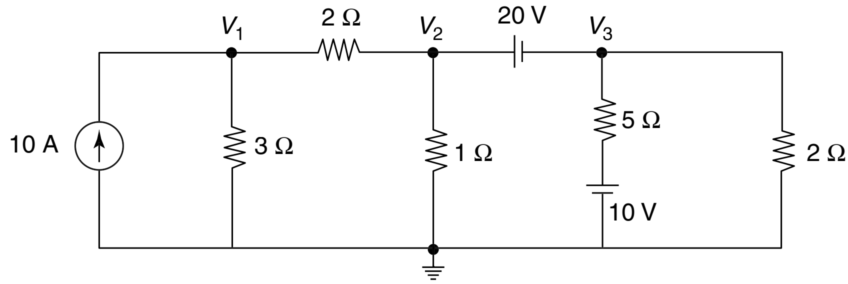

Fig. 2.118

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{3} + \frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 10$$

$$0.83\,V_1 - 0.5\,V_2 = 10 \qquad \ldots\text{(i)}$$

Nodes 2 and 3 will form a supernode.
Writing voltage equation for the supernode,

$$V_2 - V_3 = 20 \qquad \ldots\text{(ii)}$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2} + 1\right)V_2 + \left(\frac{1}{5} + \frac{1}{2}\right)V_3 = 2$$

$$-0.5\,V_1 + 1.5\,V_2 + 0.7\,V_3 = 2 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 19.04 \text{ V}$$
$$V_2 = 11.6 \text{ V}$$
$$V_3 = -8.4 \text{ V}$$

$$I_{5\,\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4 - 10}{5} = -3.68 \text{ A}$$

Example 2.84 *Find the power delivered by the 5 A current source in the network shown in Fig. 2.119.*

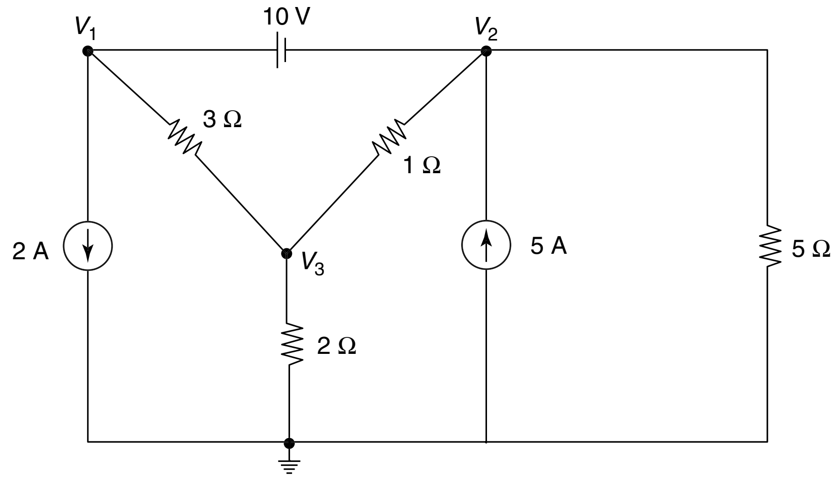

Fig. 2.119

Solution Assume that the currents are moving away from the nodes.
Nodes 1 and 2 will form a supernode.
Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \qquad \ldots\text{(i)}$$

Applying KCL at the supernode,

$$2 + \frac{V_1 - V_3}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 5$$

$$\frac{1}{3}V_1 + \left(\frac{1}{5} + 1\right)V_2 - \left(\frac{1}{3} + 1\right)V_3 = 3$$

$$0.33\,V_1 + 1.2\,V_2 - 1.33\,V_3 = 3 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$-\frac{1}{3}V_1 - V_2 + \left(\frac{1}{3} + 1 + \frac{1}{2}\right)V_3 = 0$$

$$-0.33\,V_1 - V_2 + 1.83\,V_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 13.72 \text{ V}$$

$$V_2 = 3.72 \text{ V}$$

$$V_3 = 4.51 \text{ V}$$

Power delivered by the 5 A source = $5\,V_2 = 5 \times 3.72 = 18.6$ W

Example 2.85 *In the network of Fig. 2.120, find the node voltages V_1, V_2 and V_3.*

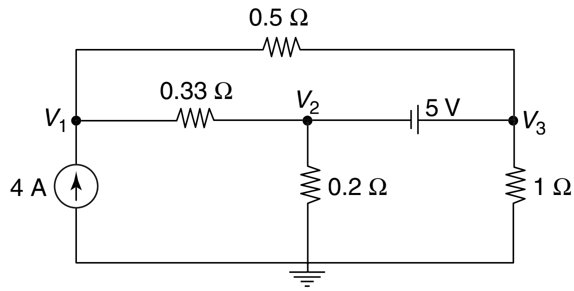

Fig. 2.120

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$4 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.5}$$

$$\left(\frac{1}{0.33} + \frac{1}{0.5}\right)V_1 - \frac{1}{0.33}V_2 - \frac{1}{0.5}V_3 = 4$$

$$5.03\,V_1 - 3.03\,V_2 - 2\,V_3 = 4 \qquad \ldots\text{(i)}$$

Nodes 2 and 3 will form a supernode.
Writing voltage equation for the supernode,

$$V_3 - V_2 = 5 \qquad \ldots\text{(ii)}$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{0.33} + \frac{V_2}{0.2} + \frac{V_3}{1} + \frac{V_3 - V_1}{0.5} = 0$$

$$\left(-\frac{1}{0.33} - \frac{1}{0.5}\right)V_1 + \left(\frac{1}{0.33} + \frac{1}{0.2}\right)V_2 + \left(1 + \frac{1}{0.5}\right)V_3 = 0$$

$$-5.03\,V_1 + 8.03\,V_2 + 3\,V_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 2.62\text{ V}$$
$$V_2 = -0.17\text{ V}$$
$$V_3 = 4.83\text{ V}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.86 *For the network shown in Fig. 2.121, determine the voltage V_x.*

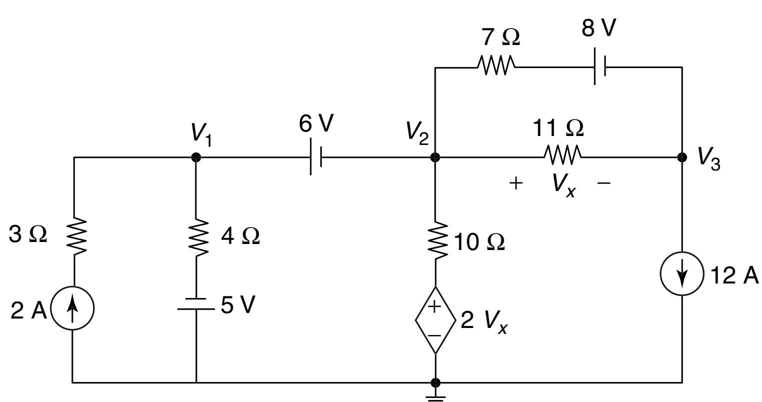

Fig. 2.121

Solution From Fig. 2.121,

$$V_x = V_2 - V_3$$

Assume that the currents are moving away from the nodes.
Node 1 and 2 will form a supernode.
Writing voltage equations for the supernode,

$$V_1 - V_2 = 6 \qquad \ldots\text{(i)}$$

Applying KCL at the supernode,

$$2 = \frac{V_1+5}{4} + \frac{V_2 - 2V_x}{10} + \frac{V_2 - 8 - V_3}{7} + \frac{V_2 - V_3}{11}$$

$$2 = \frac{V_1+5}{4} + \frac{V_2 - 2(V_2 - V_3)}{10} + \frac{V_2 - 8 - V_3}{7} + \frac{V_2 - V_3}{11}$$

$$\frac{1}{4}V_1 + \left(\frac{1}{10} - \frac{1}{5} + \frac{1}{7} + \frac{1}{11}\right)V_2 + \left(\frac{1}{5} - \frac{1}{7} - \frac{1}{11}\right)V_3 = 2 - \frac{5}{4} + \frac{8}{7}$$

$$0.25\,V_1 + 0.133\,V_2 - 0.033\,V_3 = 1.89 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3 - V_2}{11} + \frac{V_3 + 8 - V_2}{7} + 12 = 0$$

$$\left(-\frac{1}{11} - \frac{1}{7}\right)V_2 + \left(\frac{1}{11} + \frac{1}{7}\right)V_3 = -12 - \frac{8}{7}$$

$$-0.233\,V_2 + 0.233\,V_3 = -13.14 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 1.8\text{ V}$$
$$V_2 = -4.2\text{ V}$$
$$V_3 = -60.6\text{ V}$$

Example 2.87 *Find the node voltages in the network shown in Fig. 2.122.*

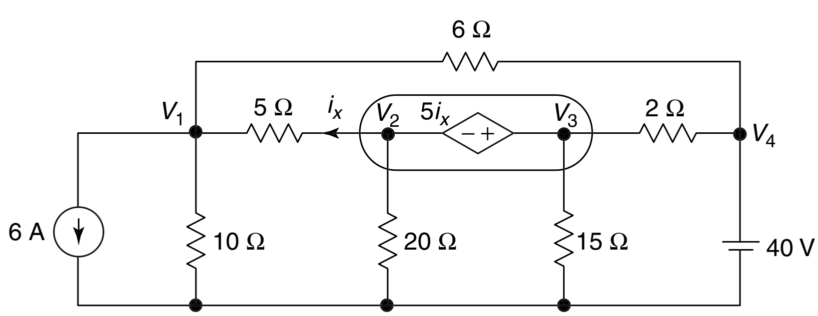

Fig. 2.122

Solution From Fig. 2.122,

$$I_x = \frac{V_2 - V_1}{5} \qquad \ldots\text{(i)}$$

For Node 4,

$$V_4 = 40 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 1,

$$6+\frac{V_1}{10}+\frac{V_1-V_2}{5}+\frac{V_1-V_4}{6}=0$$

$$6+\frac{V_1}{10}+\frac{V_1-V_2}{5}+\frac{V_1-40}{6}=0$$

$$\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{6}\right)V_1-\frac{1}{5}V_2=\frac{40}{6}-6$$

$$\frac{7}{15}V_1-\frac{1}{5}V_2=\frac{2}{3} \qquad \ldots\text{(iii)}$$

Nodes 2 and 3 will form a supernode,
Writing voltage equation for the supernode,

$$V_3-V_2=5\,I_x=5\left(\frac{V_2-V_1}{5}\right)=V_2-V_1$$

$$V_1-2V_2+V_3=0 \qquad \ldots\text{(iv)}$$

Applying KCL to the supernode,

$$\frac{V_2-V_1}{5}+\frac{V_2}{20}+\frac{V_3}{15}+\frac{V_3-V_4}{2}=0$$

$$\frac{V_2-V_1}{5}+\frac{V_2}{20}+\frac{V_3}{15}+\frac{V_3-40}{2}=0$$

$$-\frac{1}{5}V_1+\left(\frac{1}{5}+\frac{1}{20}\right)V_2+\left(\frac{1}{15}+\frac{1}{2}\right)V_3=20$$

$$-\frac{1}{5}V_1+\frac{1}{4}V_2+\frac{17}{30}V_3=20 \qquad \ldots\text{(v)}$$

Solving Eqs (iii), (iv) and (v),

$$V_1=10\text{ V}$$
$$V_2=20\text{ V}$$
$$V_3=30\text{ V}$$
$$V_4=40\text{ V}$$

Example 2.88 *Find the node voltages in the network shown in Fig. 2.123.*

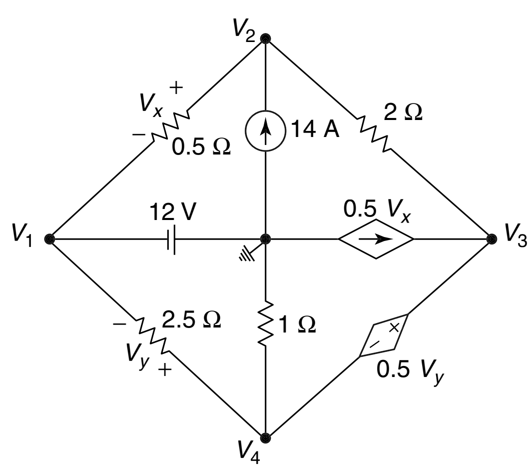

Fig. 2.123

Solution Selecting the central node as reference node,

$$V_1 = -12\text{ V} \quad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14$$

$$-\frac{1}{0.5}V_1 + \left(\frac{1}{0.5} + \frac{1}{2}\right)V_2 - \frac{1}{2}V_3 = 14$$

$$-2\,V_1 + 2.5\,V_2 - 0.5\,V_3 = 14 \quad \text{...(ii)}$$

Nodes 3 and 4 will form a supernode,
Writing voltage equation for the supernode,

$$V_3 - V_4 = 0.2V_y = 0.2(V_4 - V_1)$$

$$0.2V_1 + V_3 - 1.2V_4 = 0 \quad \text{...(iii)}$$

Applying KCL to the supernode,

$$\frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0$$

$$\frac{V_3 - V_2}{2} - 0.5(V_2 - V_1) + V_4 + \frac{V_4 - V_1}{2.5} = 0$$

$$\left(0.5 - \frac{1}{2.5}\right)V_1 - \left(\frac{1}{2} + 0.5\right)V_2 + \frac{1}{2}V_3 + \left(1 + \frac{1}{2.5}\right)V_4 = 0$$

$$0.1\,V_1 - V_2 + 0.5\,V_3 + 1.4\,V_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (i), (ii), (iii) and (iv),

$$V_1 = -12\text{ V}$$
$$V_2 = -4\text{ V}$$
$$V_3 = 0$$
$$V_4 = -2\text{ V}$$

Exercises

KIRCHHOFF'S LAWS

2.1 Find I_x and V_x in the network shown in Fig. 2.124.

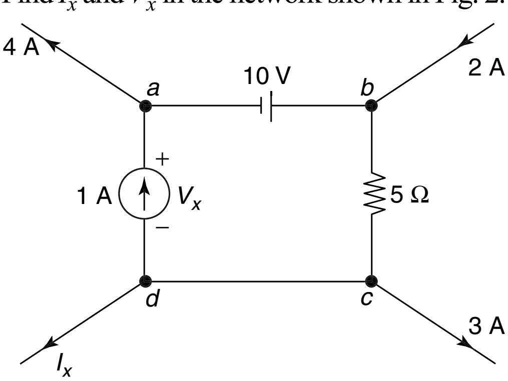

Fig. 2.124

[−5 A, −15 V]

2.2 Find V_1 and V_2 in the network shown in Fig. 2.125.

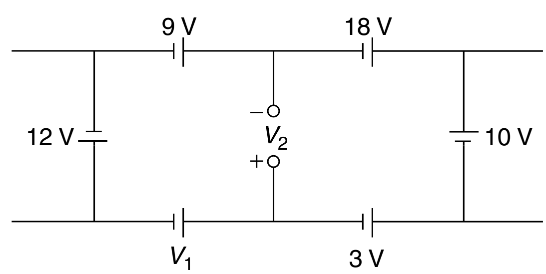

Fig. 2.125

[2 V, 5 V]

2.3 Find the values of unknown currents in the network shown in Fig. 2.126.

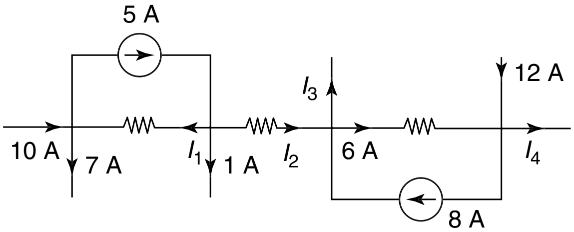

Fig. 2.126

$[I_1 = 2\text{ A}, I_2 = 2\text{ A}, I_3 = 4\text{ A}, I_4 = 10\text{ A}]$

2.4 Find the current in the branch *XY* as shown in Fig. 2.127.

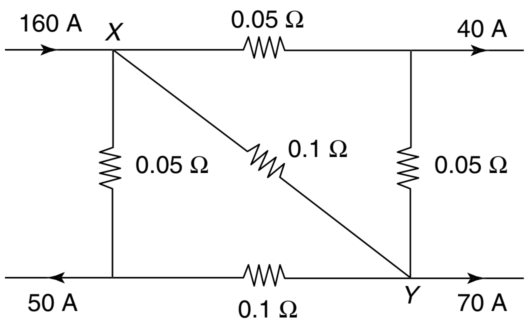

Fig. 2.127

[40 A]

2.5 Find I and V_{AB} for the network as shown in Fig. 2.128.

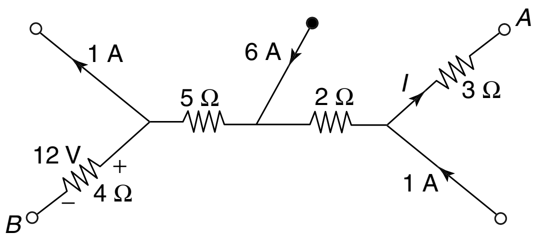

Fig. 2.128

[3 A, 19 V]

2.6 In the network shown in Fig. 2.129, find the voltage between points *A* and *B*.

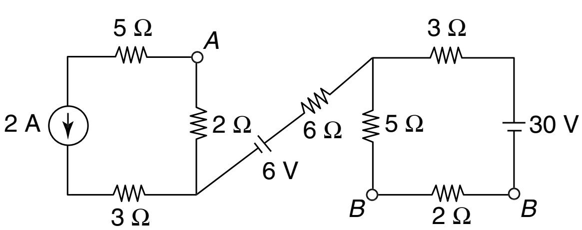

Fig. 2.129

[5 V]

2.7 In the network shown in Fig. 2.130, find the voltage between points *A* and *B*.

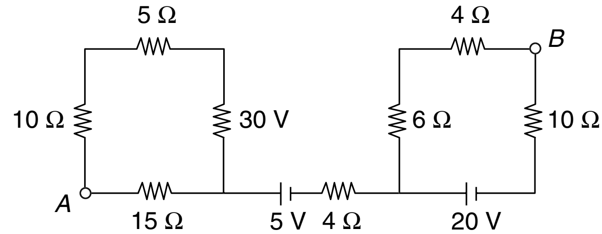

Fig. 2.130

[30 V]

MESH ANALYSIS

2.8 Find the current through the 10 Ω resistor in the network shown in Fig. 2.131.

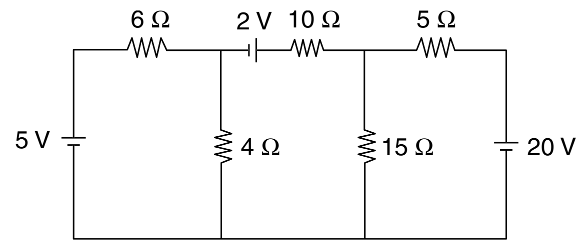

Fig. 2.131

[0.68 A]

2.9 Find the current through the 20 Ω resistor in the network shown in Fig. 2.132.

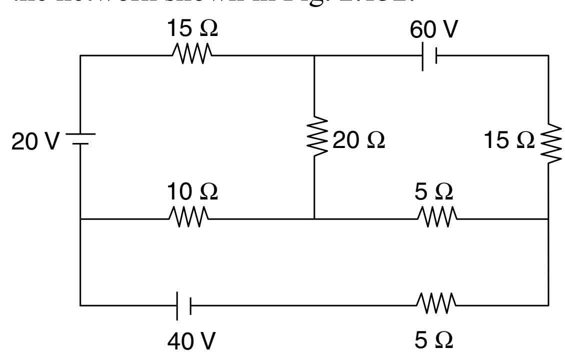

Fig. 2.132

[1.46 A]

2.10 Find the current through the 10 Ω resistor in the network shown in Fig. 2.133.

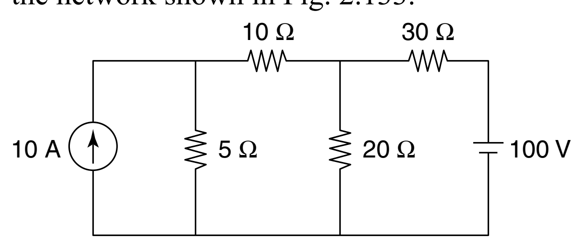

Fig. 2.133

[0.37 A]

2.11 Find the current through the 1 Ω resistor in the network shown in Fig. 2.134.

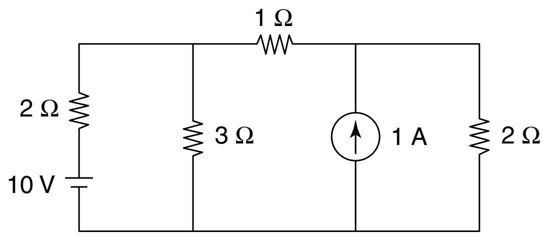

Fig. 2.134

[0.95 A]

2.12 Find the current through the 4 Ω resistor in the network shown in Fig. 2.135.

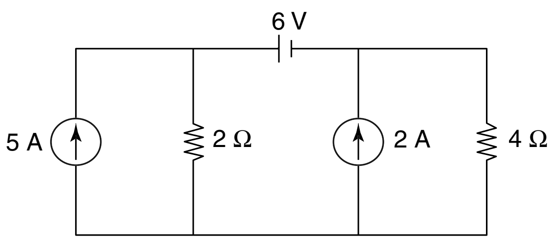

Fig. 2.135

[1.33 A]

2.13 Find currents I_x and I_y in the network shown in Fig. 2.136.

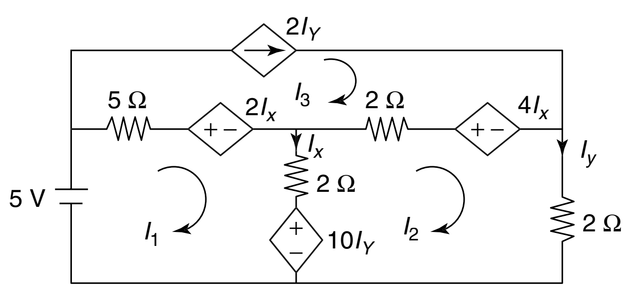

Fig. 2.136

[0.5 A, 0.1 A]

2.14 In the network shown in Fig. 2.137, find V_3 if element A is a

(i) short circuit
(ii) 5 Ω resistor
(iii) 20 V independent voltage source, positive reference on the right
(iv) dependent voltage source of 1.5 i_1, with positive reference on the right
(v) dependent current source 5 i_1, arrow directed to the right

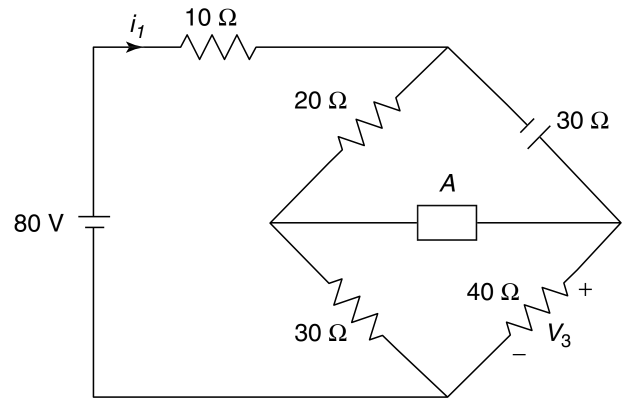

Fig. 2.137

[69.4 V, 72.38 V, 73.68 V, 70.71 V, 97.39 V]

2.15 Find currents I_1, I_2, and I_3 in the network shown in Fig. 2.138.

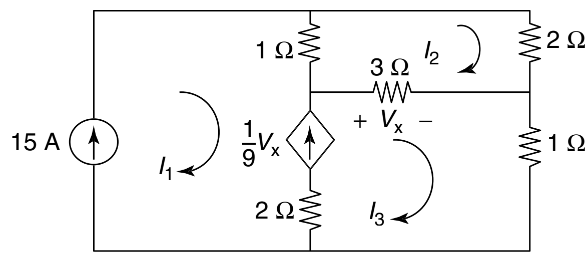

Fig. 2.138

[15 A, 11 A, 17 A]

2.16 Find currents I_x in the network shown in Fig. 2.139.

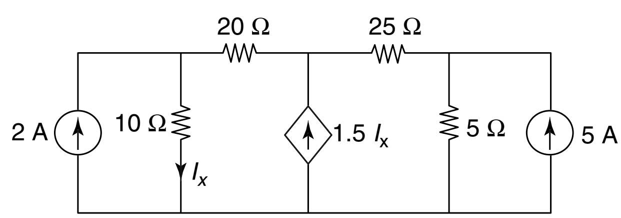

Fig. 2.139

[8.33 A]

2.17 Find currents I_1 in the network shown in Fig. 2.140.

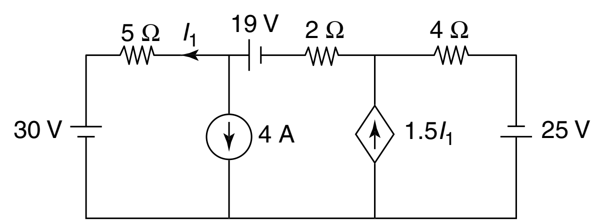

Fig. 2.140

[−12 A]

NODE ANALYSIS

2.18 Find the current I_x in the network shown in Fig. 2.141.

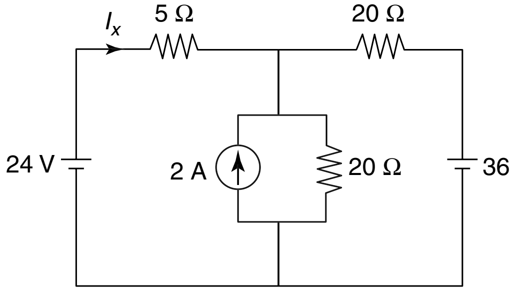

Fig. 2.141

[–0.93 A]

2.19 Find V_A and V_B in the network shown in Fig. 2.142.

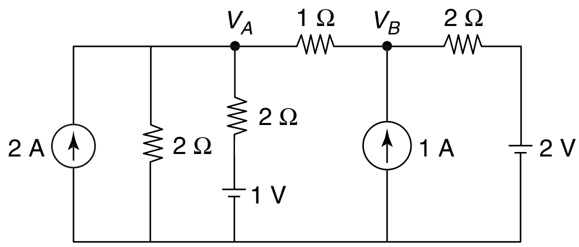

Fig. 2.142

[2.88 V,3.25 V]

2.20 Find the current through the 6 Ω resistor in the network shown in Fig. 2.143.

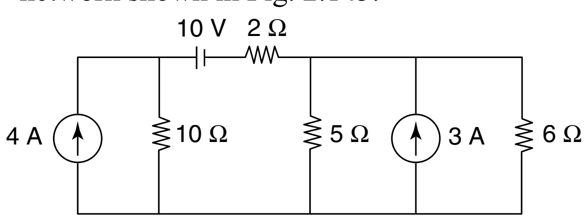

Fig. 2.143

[2.04 A]

2.21 Calculate the current through the 10 Ω resistor in the network shown in Fig. 2.144.

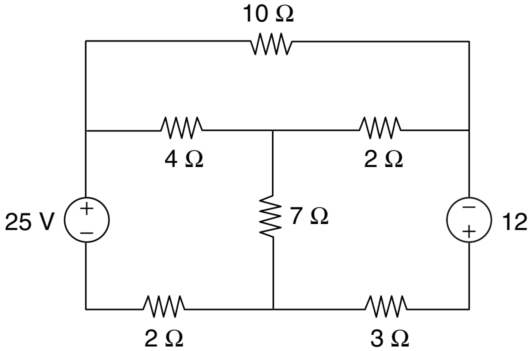

Fig. 2.144

[1.62 A]

2.22 Find the current through the branch *ab* in the network shown in Fig. 2.145.

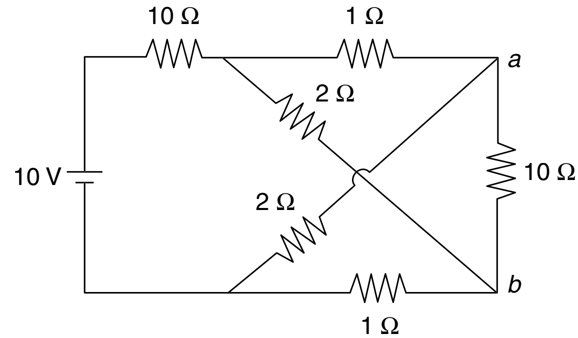

Fig. 2.145

[0.038 A]

2.23 Find the current through the 4 Ω resistor in the network shown in Fig. 2.146.

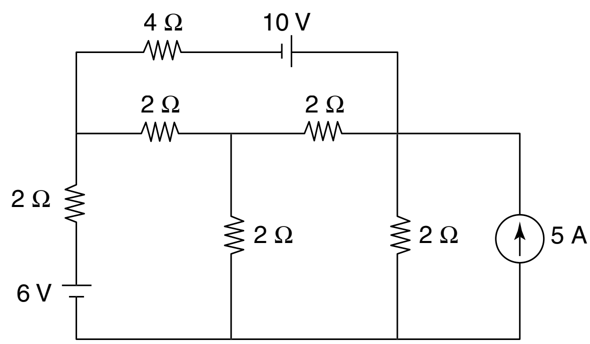

Fig. 2.146

[1.34 A]

2.24 Find the current through the 4 Ω resistor in the network shown in Fig. 2.147.

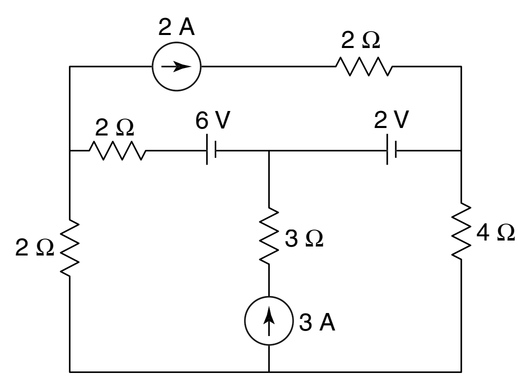

Fig. 2.147

[1 A]

2.25 Find the voltage V_x in the network shown in Fig. 2.148.

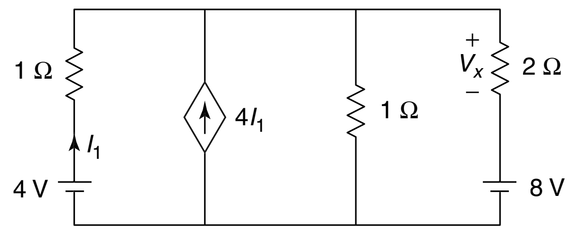

Fig. 2.148

[−4.31 V]

2.26 Find the currents V_x in the network shown in Fig. 2.149.

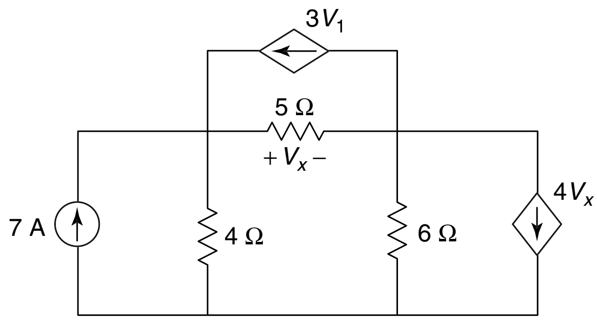

Fig. 2.149

[2.09 V]

2.27 Find the voltage V_x in the network shown in Fig. 2.150.

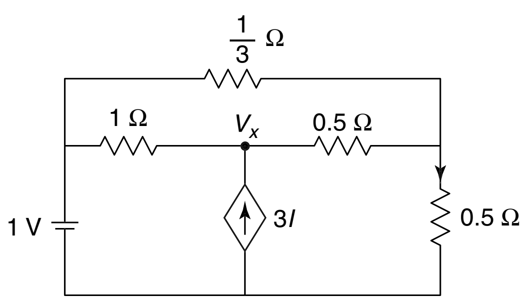

Fig. 2.150

[6.2 V]

2.28 Determine V_1 in the network shown in Fig. 2.151.

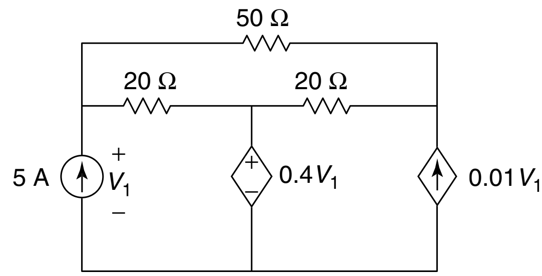

Fig. 2.151

[140 V]

2.29 Find the voltage V_y in the network shown in Fig. 2.152.

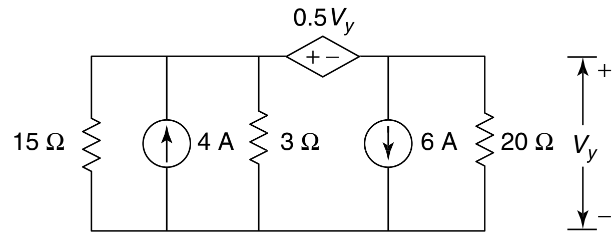

Fig. 2.152

[−10 V]

2.30 Find the voltage V_2 in the network shown in Fig. 2.153.

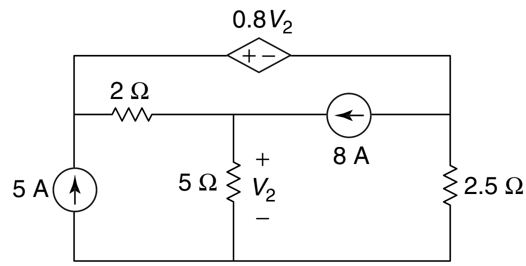

Fig. 2.153

[25.9 V]

Objective–Type Questions

2.1 Two electrical sub-networks N_1 and N_2 are connected through three resistors as shown in Fig. 2.154. The voltages across the 5 Ω resistor and 1 Ω resistor are given to be 10 V and 5 V respectively. Then the voltage across the 15 Ω resistor is

(a) −105 V (b) 105 V
(c) −15 V (d) 15 V

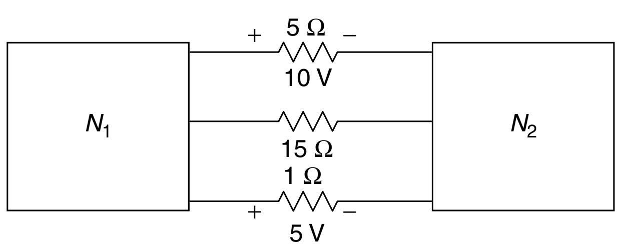

Fig. 2.154

2.2 The nodal method of circuit analysis is based on

(a) KVL and Ohm's law
(b) KCL and Ohm's law
(c) KCL and KVL
(d) KCL, KVL and Ohm's law

2.3 The voltage across terminals a and b in Fig. 2.155 is

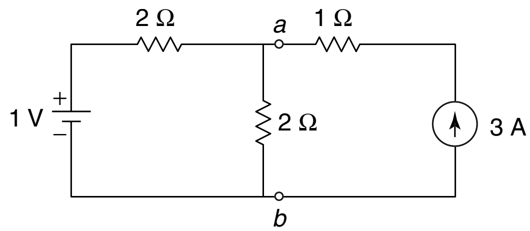

Fig. 2.155

(a) 0.5 V (b) 3 V
(c) 3.5 V (d) 4 V

2.4 The voltage V_0 in Fig. 2.156 is

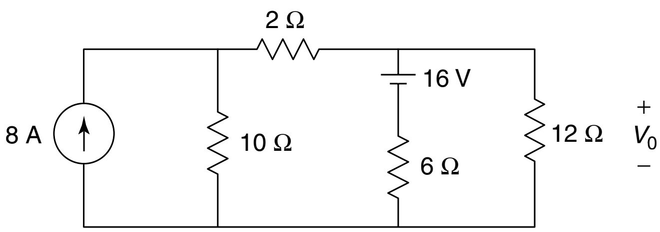

Fig. 2.156

(a) 48 V (b) 24 V
(c) 36 V (d) 28 V

2.5 The dependent current source shown in Fig. 2.157.

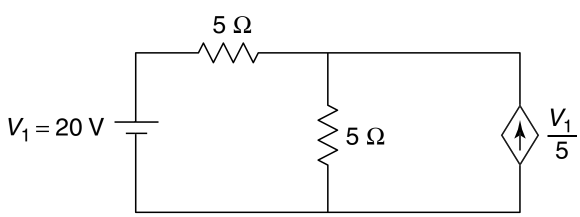

Fig. 2.157

(a) delivers 80 W
(b) absorbs 80 W
(c) delivers 40 W
(d) absorbs 40 W

2.6 If $V = 4$ in Fig. 2.158, the value of I_s is given by

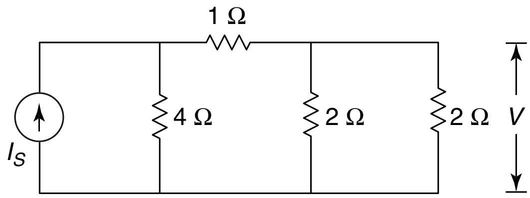

Fig. 2.158

(a) 6 A (b) 2.5 A
(c) 12 A (d) none of these

2.7 The value of V_x, V_y and V_z in Fig. 2.159 shown are

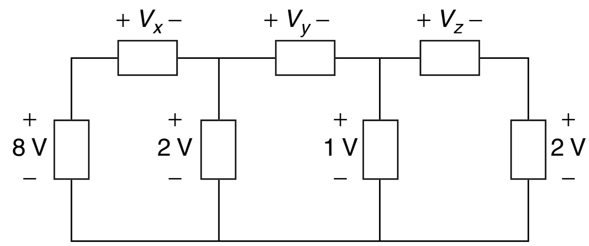

Fig. 2.159

(a) −6, 3, −3 (b) −6, −3, 1
(c) 6, 3, 3 (d) 6, 1, 3

2.8 The circuit shown in Fig. 2.160 is equivalent to a load of

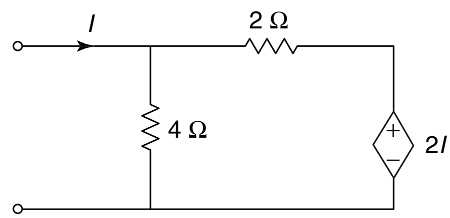

Fig. 2.160

(a) $\frac{4}{3}\ \Omega$ (b) $\frac{8}{3}\ \Omega$
(b) 4 Ω (d) 2 Ω

2.9 In the network shown in Fig. 2.161, the effective resistance faced by the voltage source is

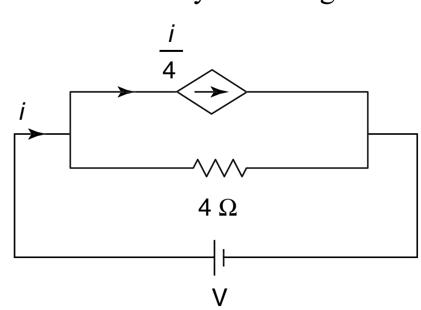

Fig. 2.161

(a) 4 Ω (b) 3 Ω

(c) 2 Ω (d) 1 Ω

2.10 A network contains only an independent current source and resistors. If the values of all resistors are doubled, the value of the node voltages will

(a) become half
(b) remain unchanged
(c) become double
(d) none of these

Answers to Objective-Type Questions

2.1. (a)	2.2. (b)	2.3. (c)	2.4. (d)	2.5. (a)	2.6. (d)	2.7. (a)
2.8. (a)	2.9. (d)	2.10. (b)	2.11. (b)	2.12. (b)	2.13. (c)	2.14. (b)

Example 6.2 *Find mesh current I_1, I_2 and I_3 in the network of Fig. 6.2.*

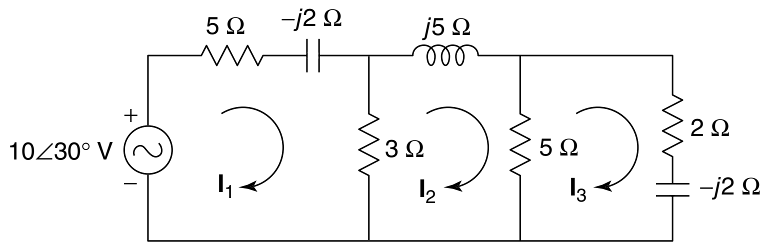

Fig. 6.2

Solution Applying KVL to Mesh 1,

$$10\angle 30^\circ - (5 - j2)\,\mathbf{I}_1 - 3\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(8 - j2)\,\mathbf{I}_1 - 3\mathbf{I}_2 = 10\angle 30^\circ \qquad \text{... (i)}$$

Applying KVL to Mesh 2,

$$-3\,(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 - 5\,(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-3\mathbf{I}_1 + (8 + j5)\,\mathbf{I}_2 - 5\mathbf{I}_3 = 0 \qquad \text{... (ii)}$$

Applying KVL to Mesh 3,

$$-5\,(\mathbf{I}_3 - \mathbf{I}_2) - (2 - j2)\,\mathbf{I}_3 = 0$$

$$-5\mathbf{I}_2 + (7 - j2)\,\mathbf{I}_3 = 0 \qquad \text{... (iii)}$$

Writing Eqs (i), (ii) and (iii),

$$\begin{bmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 30^\circ & -3 & 0 \\ 0 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 1.43\angle 38.7^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 8-j2 & 10\angle 30^\circ & 0 \\ -3 & 0 & -5 \\ 0 & 0 & 7-j2 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 0.693\angle -2.2^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 8-j2 & -3 & 10\angle 30^\circ \\ -3 & 8+j5 & 0 \\ 0 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 0.476\angle 13.8^\circ \text{ A}$$

Example 6.3 *In the network of Fig. 6.3, find the value of V_2 so that the current through (2 + j3) ohm impedance is zero.*

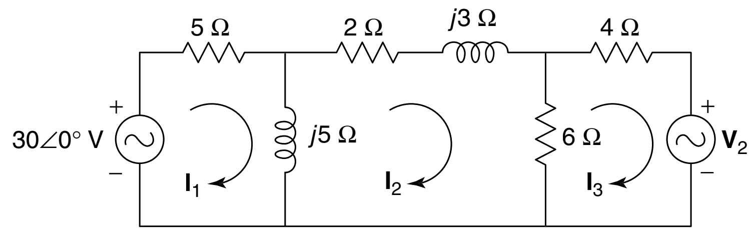

Fig. 6.3

Solution Applying KVL to Mesh 1,

$$30\angle 0° - 5\mathbf{I}_1 - j5(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(5 + j5)\mathbf{I}_1 - j5\mathbf{I}_2 = 30\angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j5(\mathbf{I}_2 - \mathbf{I}_1) - (2 + j3)\mathbf{I}_2 - 6(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-j5\mathbf{I}_1 + (8 + j8)\mathbf{I}_2 - 6\mathbf{I}_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(\mathbf{I}_3 - \mathbf{I}_2) - 4\mathbf{I}_3 - \mathbf{V}_2 = 0$$

$$-6\mathbf{I}_2 + 10\mathbf{I}_3 = -\mathbf{V}_2 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0° \\ 0 \\ -\mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5+j5 & 30\angle 0° & 0 \\ -j5 & 0 & -6 \\ 0 & -\mathbf{V}_2 & 10 \end{vmatrix}}{\begin{vmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{vmatrix}} = 0$$

$$(5 + j5)(-6\mathbf{V}_2) - (30)(-j50) = 0$$

$$\mathbf{V}_2 = \frac{j1500}{30 + j30} = 35.36\angle 45° \text{ V}$$

Example 6.4 *Find the value of the current I_3 in the network shown in Fig. 6.4.*

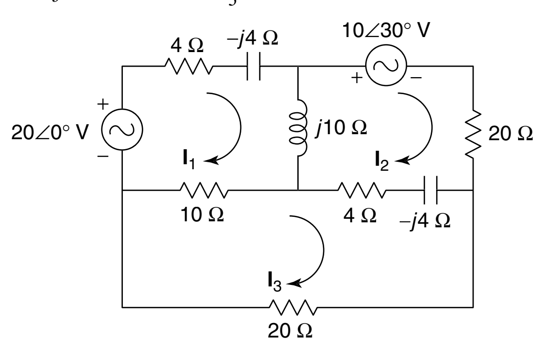

Fig. 6.4

Solution Applying KVL to Mesh 1,

$$20\angle 0° - (4 - j4)\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) - 10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$(14 + j6)\mathbf{I}_1 - j10\mathbf{I}_2 - 10\mathbf{I}_3 = 20\angle 0° \quad \text{... (i)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) - 10\angle 30° - 20\mathbf{I}_2 - (4 - j4)(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-j10\mathbf{I}_1 + (24 + j6)\mathbf{I}_2 - (4 - j4)\mathbf{I}_3 = -10\angle 30° \quad \text{... (ii)}$$

Applying KVL to Mesh 3,

$$-10(\mathbf{I}_3 - \mathbf{I}_1) - (4 - j4)(\mathbf{I}_3 - \mathbf{I}_2) - 20\mathbf{I}_3 = 0$$

$$-10\mathbf{I}_1 - (4 - j4)\mathbf{I}_2 + (34 - j4)\mathbf{I}_3 = 0 \quad \text{... (iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 14+j6 & -j10 & -10 \\ -j10 & 24+j6 & -(4-j4) \\ -10 & -(4-j4) & 34-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20\angle 0° \\ -10\angle 30° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 14+j6 & -j10 & 20\angle 0° \\ -j10 & 24+j6 & -10\angle 30° \\ -10 & -(4-j4) & 0 \end{vmatrix}}{\begin{vmatrix} 14+j6 & -10 & -10 \\ -j10 & 24+j6 & -(4-j4) \\ -10 & -(4-j4) & 34-j4 \end{vmatrix}} = 0.44\angle -14° \text{ A}$$

Example 6.5 *Find the voltage V_{AB} in the network of Fig. 6.5.*

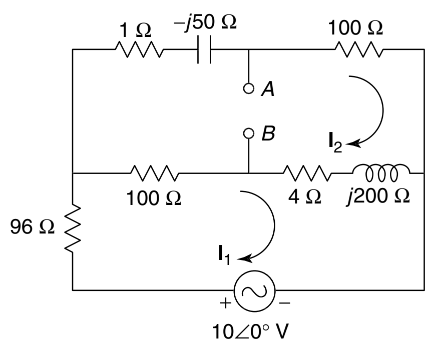

Fig. 6.5

Solution Applying KVL to Mesh 1,

$$-96\,\mathbf{I}_1 - (100 + 4 + j200)\,(\mathbf{I}_1 - \mathbf{I}_2) + 10\angle 0° = 0$$

$$(200 + j200)\,\mathbf{I}_1 - (104 + j200)\,\mathbf{I}_2 = 10\angle 0° \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(1 - j50 - 100)\,\mathbf{I}_2 - (100 + 4 + j200)\,(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$-(104 + j200)\,\mathbf{I}_1 + (205 + j150)\,\mathbf{I}_2 = 0 \quad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -(104+j200) \\ 0 & 205+j150 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}} = 0.051\angle 2.72\times 10^{-3}\,^\circ\text{A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 200+j200 & 10\angle 0^\circ \\ -(104+j200) & 0 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}} = 0.045\angle 26.34^\circ \text{ A}$$

$$\begin{aligned} \mathbf{V}_{AB} &= 100\mathbf{I}_2 - (4+j200)(\mathbf{I}_1 - \mathbf{I}_2) \\ &= 100(0.045\angle 26.34^\circ) - (4+j200)(0.051\angle 2.72\times 10^{-3}\,^\circ - 0.045\angle 26.34^\circ) \\ &= 0.058\angle -92.65^\circ \text{ V} \end{aligned}$$

Example 6.6 *For the network shown in Fig. 6.6, find the voltage across the capacitor.*

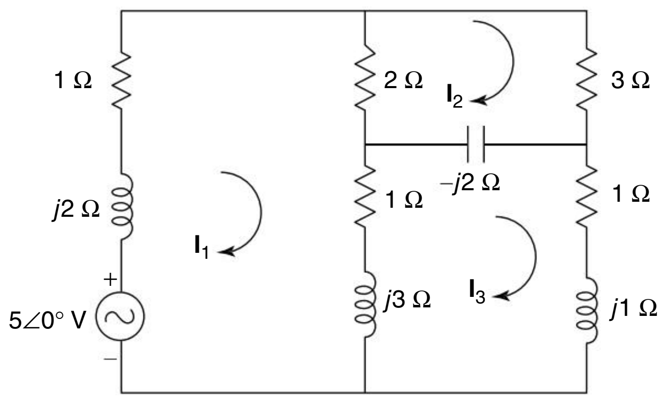

Fig. 6.6

Solution Applying KVL to Mesh 1,

$$5\angle 0^\circ - (1+j2)\mathbf{I}_1 - 2(\mathbf{I}_1 - \mathbf{I}_2) - (1+j3)(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$(4+j5)\mathbf{I}_1 - 2\mathbf{I}_2 - (1+j3)\mathbf{I}_3 = 5\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-2(\mathbf{I}_2 - \mathbf{I}_1) - 3\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathrm{I}_3) = 0$$

$$-2\mathbf{I}_1 + (5-j2)\mathbf{I}_2 + j2\mathbf{I}_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-(1+j3)(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) - (1+j1)\mathbf{I}_3 = 0$$

$$-(1+j3)\mathbf{I}_1 + j2\mathbf{I}_2 + (2+j2)\mathbf{I}_3 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 5\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4+j5 & 5\angle 0^\circ & -(1+j3) \\ -2 & 0 & j2 \\ -(1+j3) & 0 & 2+j2 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{vmatrix}} = 0.65\angle 130.51^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 4+j5 & -2 & 5\angle 0^\circ \\ -2 & 5-j2 & 0 \\ -(1+j3) & j2 & 0 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{vmatrix}} = 0.91\angle -21.51^\circ \text{ A}$$

$$\mathbf{V}_c = (-j2)(\mathbf{I}_3 - \mathbf{I}_2) = (-j2)(0.91\angle -21.51^\circ - 0.65\angle 130.51^\circ) = 3.03\angle -123.12^\circ \text{ V}$$

Example 6.7 *Find the voltage across the 2 Ω resistor in the network of Fig. 6.7.*

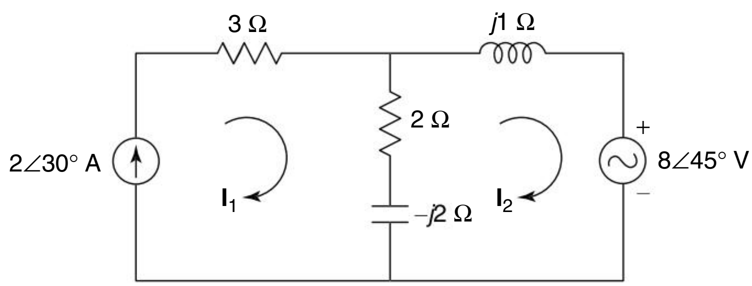

Fig. 6.7

Solution For Mesh 1,

$$\mathbf{I}_1 = 2\angle 30^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(2-j2)(\mathbf{I}_2 - \mathbf{I}_1) - j1\mathbf{I}_2 - 8\angle 45^\circ = 0$$

$$(2-j2)\mathbf{I}_1 - (2-j1)\mathbf{I}_2 = 8\angle 45^\circ \qquad \text{...(ii)}$$

Substituting $\mathbf{I}_1$ in Eq. (i),

$$(2-j2)(2\angle 30^\circ) - (2-j1)\mathbf{I}_2 = 8\angle 45^\circ$$

$$\mathbf{I}_2 = \frac{-(8\angle 45^\circ) + (2-j2)(2\angle 30^\circ)}{2-j1} = 3.19\angle -65^\circ \text{ A}$$

$$\mathbf{V}_{2\,\Omega} = 2(\mathbf{I}_1 - \mathbf{I}_2) = 2(2\angle 30^\circ - 3.19\angle -65^\circ) = 7.82\angle 84.37^\circ \text{ V}$$

Example 6.8 *Find the current through 3 Ω resistor in the network of Fig. 6.8.*

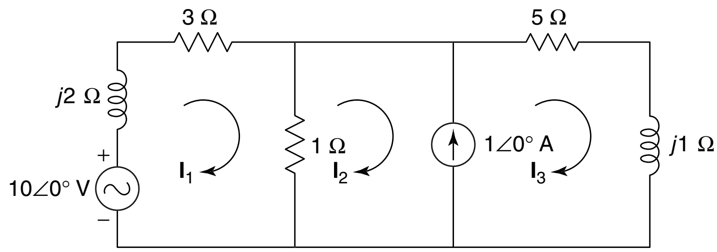

Fig. 6.8

Solution Applying KVL to Mesh 1,

$$10\angle 0^\circ - j2\mathbf{I}_1 - 3\mathbf{I}_1 - 1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(4 + j2)\mathbf{I}_1 - \mathbf{I}_2 = 10\angle 0^\circ \qquad \ldots\text{(i)}$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$\mathbf{I}_3 - \mathbf{I}_2 = 1\angle 0^\circ \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-1(\mathbf{I}_2 - \mathbf{I}_1) - 5\mathbf{I}_3 - j1\mathbf{I}_3 = 0$$

$$\mathbf{I}_1 - \mathbf{I}_2 - (5 + j1)\mathbf{I}_3 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4+j2 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & -(5+j1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 1\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -1 & 0 \\ 1\angle 0^\circ & -1 & 1 \\ 0 & -1 & -(5+j1) \end{vmatrix}}{\begin{vmatrix} 4+j2 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & -(5+j1) \end{vmatrix}} = 2.11\angle -28.01^\circ \text{ A}$$

$$\mathbf{I}_{3\,\Omega} = \mathbf{I}_1 = 2.11\angle -28.01^\circ \text{ A}$$

Example 6.9 *Find the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ in the network of Fig. 6.9.*

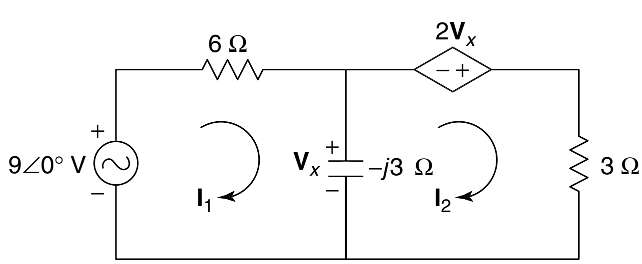

Fig. 6.9

Solution From Fig. 6.9,

$$\mathbf{V}_x = -j3(\mathbf{I}_1 - \mathbf{I}_2) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$9\angle 0° - 6\mathbf{I}_1 + j3(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(6 - j3)\mathbf{I}_1 + j3\mathbf{I}_2 = 9\angle 0° \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$j3(\mathbf{I}_2 - \mathbf{I}_1) + 2\mathbf{V}_x - 3\mathbf{I}_2 = 0$$

$$j3\mathbf{I}_2 - j3\mathbf{I}_1 + 2[-j3(\mathbf{I}_1 - \mathbf{I}_2)] - 3\mathbf{I}_2 = 0$$

$$j9\mathbf{I}_1 + (3 - j9)\mathbf{I}_2 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 6-j3 & j3 \\ j9 & 3-j9 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 9\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 9\angle 0° & j3 \\ 0 & 3-j9 \end{vmatrix}}{\begin{vmatrix} 6-j3 & j3 \\ j9 & 3-j9 \end{vmatrix}} = 1.3\angle 2.49° \text{ A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 6-j3 & 9\angle 0° \\ j9 & 0 \end{vmatrix}}{\begin{vmatrix} 6-j3 & j3 \\ j9 & 3-j9 \end{vmatrix}} = 1.24\angle -15.95° \text{ A}$$

Example 6.10 *Find the voltage across the 4 Ω resistor in the network of Fig. 6.10.*

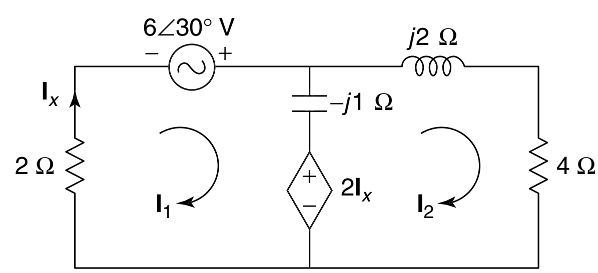

Fig. 6.10

Solution From Fig. 6.10,

$$\mathbf{I}_x = \mathbf{I}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$-2\mathbf{I}_1 + 6\angle 30° + j1(\mathbf{I}_1 - \mathbf{I}_2) - 2\mathbf{I}_x = 0$$

$$-2\mathbf{I}_1 + 6\angle 30° + j1\mathbf{I}_1 - j1\mathbf{I}_2 - 2\mathbf{I}_1 = 0$$

$$(4 - j1)\mathbf{I}_1 + j1\mathbf{I}_2 = 6\angle 30° \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$2\mathbf{I}_x + j1(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 - 4\mathbf{I}_2 = 0$$

$$2\mathbf{I}_1 + j1\mathbf{I}_2 - j1\,\mathbf{I}_1 - j2\mathbf{I}_2 - 4\mathbf{I}_2 = 0$$

$$(2 - j1)\mathbf{I}_1 - (4 + j1)\mathbf{I}_2 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4 - j1 & j1 \\ 2 - j1 & -(4 + j1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 6\angle 30^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4 - j1 & 6\angle 30^\circ \\ 2 - j1 & 0 \end{vmatrix}}{\begin{vmatrix} 4 - j1 & j1 \\ 2 - j1 & -(4 + j1) \end{vmatrix}} = 0.74\angle -2.91^\circ \text{ A}$$

$$V_{4\Omega} = 4\mathbf{I}_2 = 4(0.74\angle -2.91^\circ) = 2.96\angle -2.91^\circ \text{ V}$$

6.3 NODE ANALYSIS

Node analysis uses Kirchhoff's current law for finding currents and voltages in a network. For ac networks, Kirchhoff's current law states that the phasor sum of currents meeting at a point is equal to zero.

Example 6.11 *In the network shown in Fig. 6.11, determine $\mathbf{V}_a$ and $\mathbf{V}_b$.*

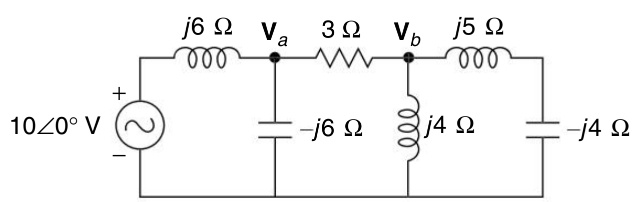

Fig. 6.11

Solution Applying KCL at Node a,

$$\frac{\mathbf{V}_a - 10\angle 0^\circ}{j6} + \frac{\mathbf{V}_a}{-j6} + \frac{\mathbf{V}_a - \mathbf{V}_b}{3} = 0$$

$$\left(\frac{1}{j6} - \frac{1}{j6} + \frac{1}{3}\right)\mathbf{V}_a - \frac{1}{3}\mathbf{V}_b = \frac{10\angle 0^\circ}{j6}$$

$$0.33\mathbf{V}_a - 0.33\mathbf{V}_b = 1.67\angle -90^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node b,

$$\frac{\mathbf{V}_b - \mathbf{V}_a}{3} + \frac{\mathbf{V}_b}{j4} + \frac{\mathbf{V}_b}{j1} = 0$$

$$-\frac{1}{3}\mathbf{V}_a + \left(\frac{1}{3} + \frac{1}{j4} + \frac{1}{j1}\right)\mathbf{V}_b = 0$$

$$-0.33\mathbf{V}_a + (0.33 - j1.25)\mathbf{V}_b = 0 \qquad \ldots\text{(ii)}$$

Adding Eqs (i) and (ii),

$$-j1.25\mathbf{V}_b = 1.67\angle -90^\circ$$

$$\mathbf{V}_b = \frac{1.67\angle -90^\circ}{-j1.25} = 1.34\angle 0^\circ \text{ V}$$

Substituting $\mathbf{V}_b$ in Eq. (i),

$$0.33\mathbf{V}_a - 0.33(1.34\angle 0^\circ) = 1.67\angle -90^\circ$$

$$\mathbf{V}_a = \frac{1.73\angle 75.17^\circ}{0.33} = 5.24\angle -75.17^\circ \text{ V}$$

Example 6.12 *For the network shown in Fig. 6.12, find the voltages V_1 and V_2.*

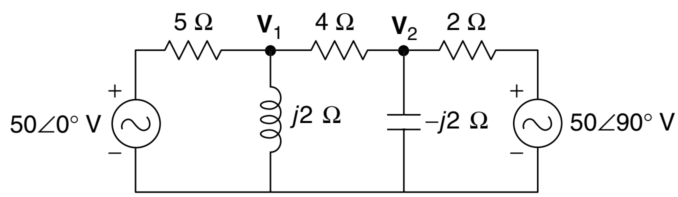

Fig. 6.12

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1 - 50\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{4} = 0$$

$$\left(\frac{1}{5} + \frac{1}{j2} + \frac{1}{4}\right)\mathbf{V}_1 - \frac{1}{4}\mathbf{V}_2 = 10\angle 0^\circ$$

$$(0.45 - j0.5)\mathbf{V}_1 - 0.25\mathbf{V}_2 = 10\angle 0^\circ \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{4} + \frac{\mathbf{V}_2}{-j2} + \frac{\mathbf{V}_2 - 50\angle 90^\circ}{2} = 0$$

$$-\frac{1}{4}\mathbf{V}_1 + \left(\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2}\right)\mathbf{V}_2 = 25\angle 90^\circ$$

$$-0.25\mathbf{V}_1 + (0.75 + j0.5)\mathbf{V}_2 = 25\angle 90^\circ \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 25\angle 90^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -0.25 \\ j25 & 0.75 + j0.5 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = 24.7\angle 72.25^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 0.45 - j0.5 & 10\angle 0^\circ \\ -0.25 & 25\angle 90^\circ \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = 34.34\angle 52.82^\circ \text{ V}$$

Example 6.13 *Find the voltage $\mathbf{V}_{AB}$ in the network of Fig. 6.13.*

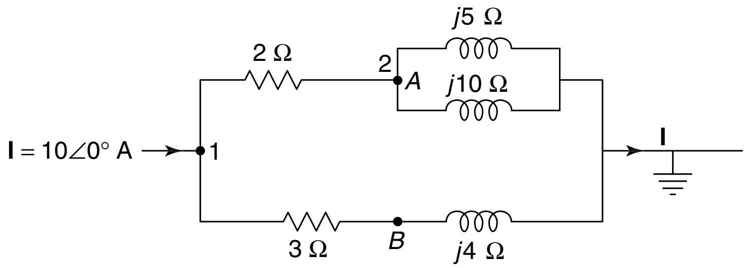

Fig. 6.13

Solution Applying KCL at Node 1,

$$10\angle 0° = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} + \frac{\mathbf{V}_1}{3+j4}$$

$$\left(\frac{1}{2} + \frac{1}{3+j4}\right)\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 = 10\angle 0°$$

$$(0.62 - j0.16)\mathbf{V}_1 - 0.5\mathbf{V}_2 = 10\angle 0° \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{2} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{j10} = 0$$

$$-\frac{1}{2}\mathbf{V}_1 + \left(\frac{1}{2} + \frac{1}{j5} + \frac{1}{j10}\right)\mathbf{V}_2 = 0$$

$$-0.5\mathbf{V}_1 + (0.5 - j0.3)\mathbf{V}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0° & -0.5 \\ 0 & 0.5 - j0.3 \end{vmatrix}}{\begin{vmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{vmatrix}} = 21.8\angle 56.42° \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 0.62 - j0.16 & 10\angle 0° \\ -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{vmatrix}} = 18.7\angle 87.42° \text{ V}$$

$$\mathbf{V}_A = \mathbf{V}_2 = 18.7\angle 87.42° \text{ V}$$

$$\mathbf{V}_B = \frac{\mathbf{V}_1}{3+j4}(j4) = \frac{21.8\angle 56.42°}{3+j4}(j4) = 17.45\angle 93.32° \text{ V}$$

$$\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B = (18.7\angle 87.42°) - (17.45\angle 93.32°) = 2.23\angle 34.1° \text{ V}$$

Example 6.14 *Find the node voltages V_1 and V_2 in the network of Fig. 6.14.*

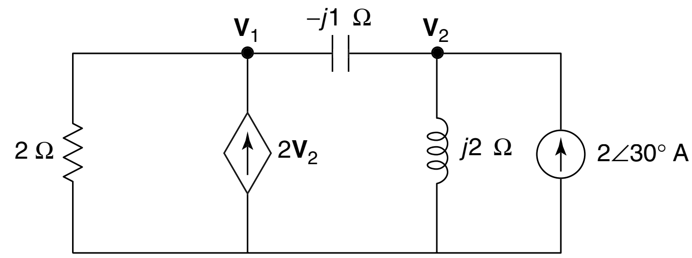

Fig. 6.14

Solution Applying KCL at Node 1,

$$\frac{\mathbf{V}_1}{2}+\frac{\mathbf{V}_1-\mathbf{V}_2}{-j1}=2\mathbf{V}_2$$

$$\left(\frac{1}{2}+\frac{1}{-j1}\right)\mathbf{V}_1-\left(2-\frac{1}{j1}\right)\mathbf{V}_2=0$$

$$(0.5+j1)\,\mathbf{V}_1-(2+j1)\mathbf{V}_2=0 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j1}+\frac{\mathbf{V}_2}{j2}=2\angle 30^\circ$$

$$\frac{1}{j1}\mathbf{V}_1+\left(\frac{1}{-j1}+\frac{1}{j2}\right)\mathbf{V}_2=2\angle 30^\circ$$

$$-j1\,\mathbf{V}_1+j\,0.5\,\mathbf{V}_2=2\angle 30^\circ \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}0.5+j1 & -(2+j1)\\ -j1 & j0.5\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}0\\ 2\angle 30^\circ\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1=\frac{\begin{vmatrix}0 & -(2+j1)\\ 2\angle 30^\circ & j0.5\end{vmatrix}}{\begin{vmatrix}0.5+j1 & -(2+j1)\\ -j1 & j0.5\end{vmatrix}}=2.46\angle 130.62^\circ \text{ V}$$

$$\mathbf{V}_2=\frac{\begin{vmatrix}0.5+j1 & 0\\ -j1 & 2\angle 30^\circ\end{vmatrix}}{\begin{vmatrix}0.5+j1 & -(2+j1)\\ -j1 & j0.5\end{vmatrix}}=1.23\angle 167.49^\circ \text{ V}$$

Example 6.15 *In the network of Fig 6.15, find the voltage V_2 which results in zero current through 4 Ω resistor.*

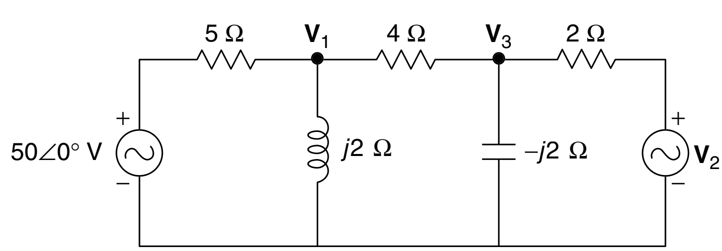

Fig. 6.15

Solution Applying KCL at Node 1,

$$\frac{\mathbf{V}_1 - 50\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{4} = 0$$

$$\left(\frac{1}{5} + \frac{1}{j2} + \frac{1}{4}\right)\mathbf{V}_1 - \frac{1}{4}\mathbf{V}_3 = 10\angle 0^\circ$$

$$(0.45 - j0.5)\mathbf{V}_1 - 0.25\mathbf{V}_3 = 10\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$\frac{\mathbf{V}_3 - \mathbf{V}_1}{4} + \frac{\mathbf{V}_3}{-j2} + \frac{\mathbf{V}_3 - \mathbf{V}_2}{2} = 0$$

$$-\frac{1}{4}\mathbf{V}_1 + \left(\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2}\right)\mathbf{V}_3 = 0.5\mathbf{V}_2$$

$$-0.25\mathbf{V}_1 + (0.75 + j0.5)\,\mathbf{V}_3 = 0.5\mathbf{V}_2 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0.5\mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -0.25 \\ 0.5\mathbf{V}_2 & 0.75 + j0.5 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = \frac{10(0.75 + j0.5) + 0.125\,\mathbf{V}_2}{0.55\angle -15.95^\circ}$$

$$\mathbf{V}_3 = \frac{\begin{vmatrix} 0.45 - j\,0.5 & 10\angle 0^\circ \\ -0.25 & 0.5\mathbf{V}_2 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = \frac{0.5\,\mathbf{V}_2(0.45 - j0.5) + 2.5}{0.55\angle -15.95^\circ}$$

$$\mathbf{I}_{4\,\Omega} = \frac{\mathbf{V}_1 - \mathbf{V}_3}{4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_3$$

$$\frac{10(0.75 + j0.5) + 0.125\mathbf{V}_2}{0.55\angle -15.95^\circ} = \frac{0.5\mathbf{V}_2(0.45 - j0.5) + 2.5}{0.55\angle -15.95^\circ}$$

$$7.5 + 0.125\mathbf{V}_2 - j\,5 = 2.5 + 0.225\mathbf{V}_2 - j0.25\mathbf{V}_2$$

$$5 + j\,5 = \mathbf{V}_2(0.1 - j0.25)$$

$$\mathbf{V}_2 = \frac{5 + j5}{0.1 - j0.25} = 26.26\angle 113.2^\circ\ \text{V}$$

Example 6.16 *Find the voltage across the capacitor in the network of Fig. 6.16.*

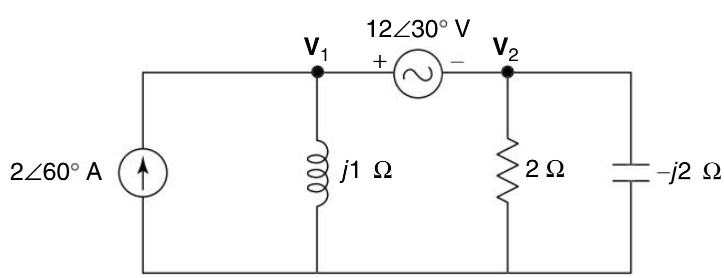

Fig. 6.16

Solution Nodes 1 and 2 will form a supernode.
Writing the voltage equation for the supernode,

$$\mathbf{V}_1 - \mathbf{V}_2 = 12\angle 30° \qquad \ldots\text{(i)}$$

Applying KCL to the supernode,

$$\frac{\mathbf{V}_1}{j1} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2}{-j2} = 2\angle 60°$$

$$(-j1)\,\mathbf{V}_1 + (0.5 + j0.5)\,\mathbf{V}_2 = 2\angle 60° \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 30° \\ 2\angle 60° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 1 & 12\angle 30° \\ -j1 & 2\angle 60° \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{vmatrix}} = 18.55\angle 157.42° \text{ V}$$

$$\mathbf{V}_c = \mathbf{V}_2 = 18.55\angle 157.42° \text{ V}$$

6.4 SUPERPOSITION THEOREM

The superposition theorem can be used to analyse an ac network containing more than one source. The superposition theorem states that *in a network containing more than one voltage source or current source, the total current or voltage in any branch of the network is the phasor sum of currents or voltages produced in that branch by each source acting separately*. As each source is considered, all of the other sources are replaced by their internal impedances. This theorem is valid only for linear systems.

Example 6.17 *Find the current through the 3 + j4 ohm impedance.*

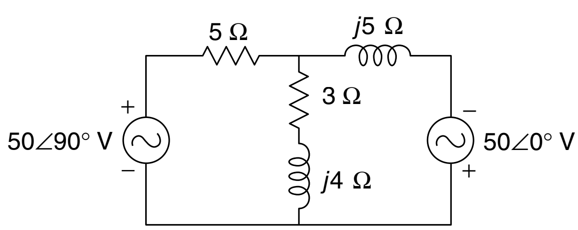

Fig. 6.17

Solution

Step I When the 50 ∠90° V source is acting alone (Fig. 6.18)

$$\mathbf{Z}_T = 5 + \frac{(3 + j4)(j5)}{3 + j9} = 6.35\angle 23.2° \ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 90°}{6.35\angle 23.2°} = 7.87\angle 66.8° \text{ A}$$

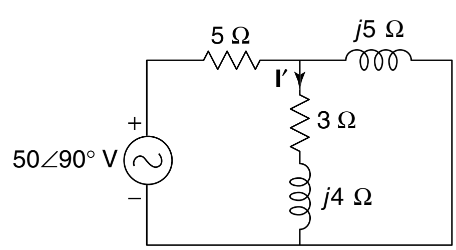

Fig. 6.18

By current division rule,

$$\mathbf{I}' = (7.87\angle 66.8°)\left(\frac{j5}{3+j9}\right) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step II When the $50\angle 0°$ V source is acting alone (Fig. 6.19)

$$\mathbf{Z}_T = j5 + \frac{5(3+j4)}{8+j4} = 6.74\angle 68.2° \ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 0°}{6.74\angle 68.2°} = 7.42\angle -68.2° \text{ A}$$

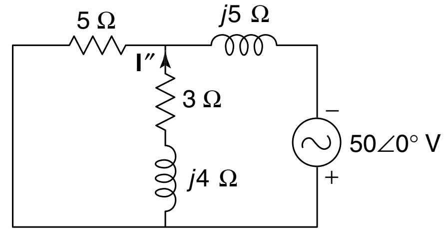

Fig. 6.19

By current division rule,

$$\mathbf{I}'' = (7.42\angle -68.2°)\left(\frac{5}{8+j4}\right) = 4.15\angle -94.77° \text{ A}(\uparrow) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 4.15\ \angle 85.3° + 4.15\ \angle 85.3° = 8.31\ \angle 85.3° \text{A}\ (\downarrow)$$

Example 6.18 *Determine the voltage across the (2 + j5) ohm impedance for the network shown in Fig.6.20.*

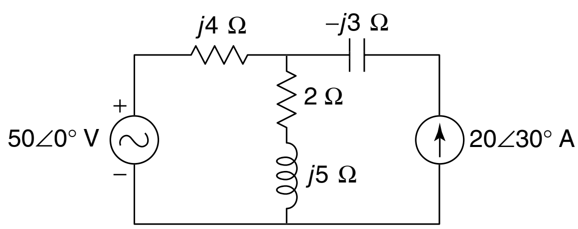

Fig. 6.20

Solution

Step I When the $50\angle 0°$ V source is acting alone (Fig. 6.21)

$$\mathbf{I} = \frac{50\angle 0°}{2+j4+j5} = 5.42\angle -77.47° \text{ A}$$

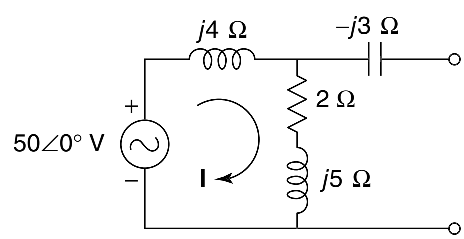

Fig. 6.21

Voltage cross $(2+j5)\ \Omega$ impedance

$$\mathbf{V}' = (2+j5)\ (5.42\ \angle -77.47°) = 29.16\ \angle -9.28° \text{ V}$$

Step II When the $20\angle 30°$ A source is acting alone (Fig. 6.22)
By current division rule,

$$\mathbf{I} = (20\angle 30°)\left(\frac{j4}{2+j9}\right) = 8.68\angle 42.53° \text{ A}$$

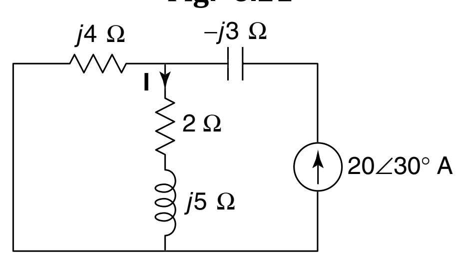

Fig. 6.22

Voltage across $(2+j5)\ \Omega$ impedance

$$\mathbf{V}'' = (2+j5)\ (8.68\ \angle 42.53°) = 46.69\ \angle 110.72° \text{ V}$$

Step III By superposition theorem,

$$\mathbf{V} = \mathbf{V}' + \mathbf{V}'' = 29.16 \angle{-9.28°} + 46.69 \angle 110.72° = 40.85 \angle 72.53° \text{ V}$$

Example 6.19 *Determine the voltage $\mathbf{V}_{AB}$ for the network shown in Fig. 6.23.*

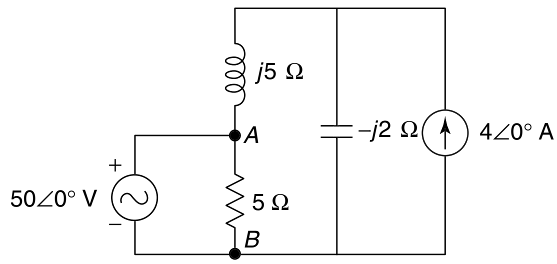

Fig. 6.23

Solution

Step I When the 50∠0° V source is acting alone (Fig. 6.24)

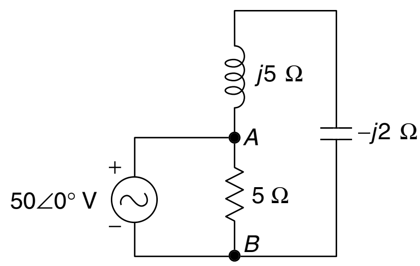

Fig. 6.24

$$\mathbf{V}'_{AB} = 50\angle 0° \text{ V}$$

Step II When the 4∠0° A source is acting alone (Fig. 6.25)

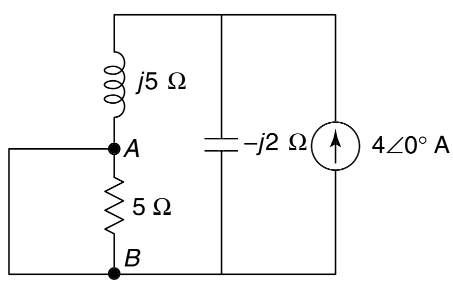

Fig. 6.25

$$\mathbf{V}''_{AB} = 0$$

Step III By superposition theorem,

$$\mathbf{V}_{AB} = \mathbf{V}'_{AB} + \mathbf{V}''_{AB} = 50\angle 0° + 0 = 50\angle 0° \text{ V}$$

Example 6.20 *Find the current **I** in the network shown in Fig. 6.26.*

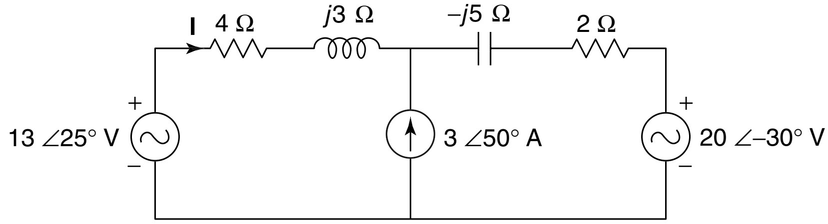

Fig. 6.26

Solution

Step I When the 13∠25° V source is acting alone (Fig. 6.27)

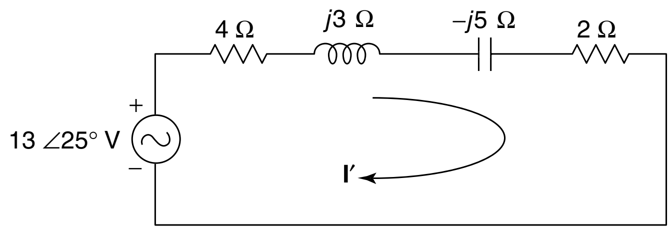

Fig. 6.27

$$\mathbf{I}' = \frac{13\angle 25^\circ}{6 - j2} = 2.057\angle 43.43^\circ \text{ A} \quad (\rightarrow)$$

Step II When the 20∠−30° V source is acting alone (Fig. 6.28)

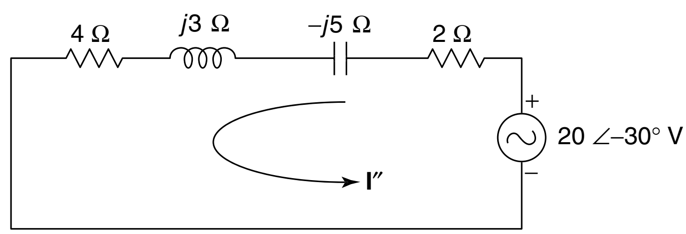

Fig. 6.28

$$\mathbf{I}'' = \frac{20\angle -30^\circ \text{ V}}{6 - j2} = 3.16\angle -11.57^\circ \text{ A}(\leftarrow) = 3.16\angle 168.43^\circ \text{ A}(\rightarrow)$$

Step III When the 3∠50° A source is acting alone (Fig. 6.29)

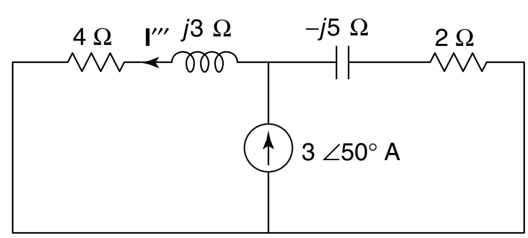

Fig. 6.29

By current division rule,

$$\mathbf{I}''' = 3\angle 50° \times \frac{2-j5}{6-j2} = 2.56\angle 0.23°\ \text{A}(\leftarrow) = 2.56\angle -179.77°\ \text{A}(\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 2.057\ \angle 43.13° + 3.16\ \angle 168.43° + 2.56\ \angle{-179.77°}\ \text{A} = 4.62\ \angle 153.99°\ \text{A}\ (\rightarrow)$$

Example 6.21 *Find the current through the j3 Ω reactance in the network of Fig 6.30.*

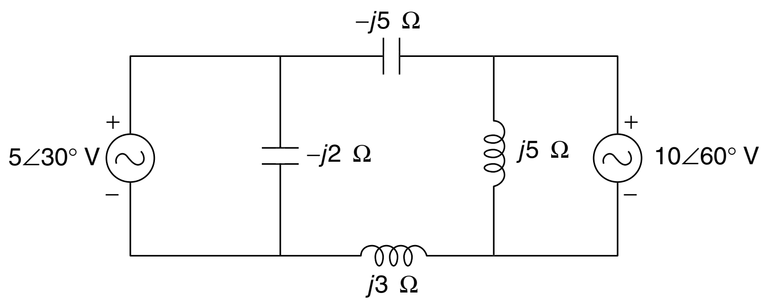

Fig. 6.30

Solution

Step I When the 5∠30° V source is acting alone (Fig. 6.31)

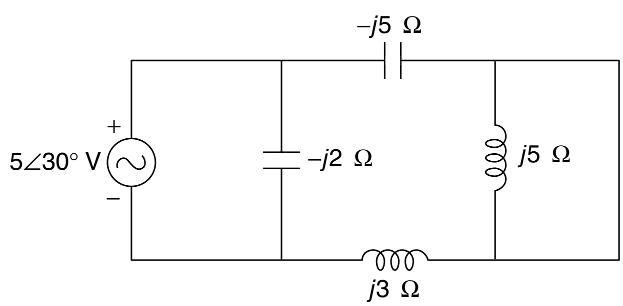

Fig. 6.31

When a short circuit is placed across $j15\ \Omega$ reactance, it gets shorted as shown in Fig 6.32.

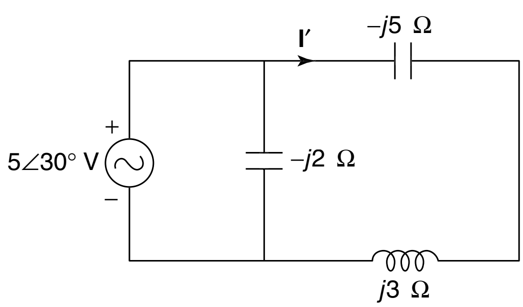

Fig. 6.32

$$\mathbf{I}' = \frac{5\angle 30°}{-j5 + j3} = 2.5\angle 120°\ \text{A}(\leftarrow)$$

Step II When the $10\angle 60°$ V source is acting alone (Fig. 6.33)

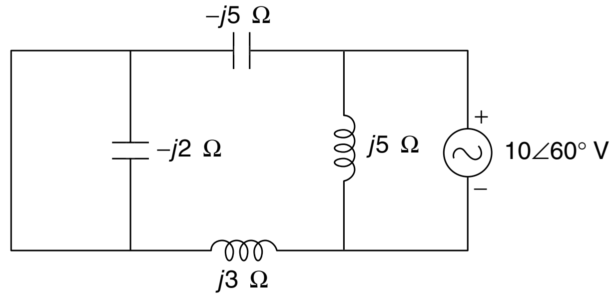

Fig. 6.33

When a short circuit is placed across the $-j2\ \Omega$ reactance, it gets shorted as shown in Fig. 6.34

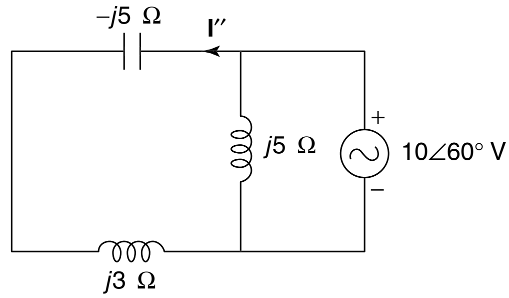

Fig. 6.34

$$\mathbf{I}'' = \frac{10\angle 60°}{-j5 + j3} = 5\angle 150° \text{ A } (\rightarrow) = 5\angle -30° \text{ A } (\leftarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 2.5\angle 120° + 5\angle -30° = 3.1\angle -6.21° \text{ A} \quad (\leftarrow)$$

Example 6.22 *Find the current $\mathbf{I}_0$ in the network of Fig. 6.35.*

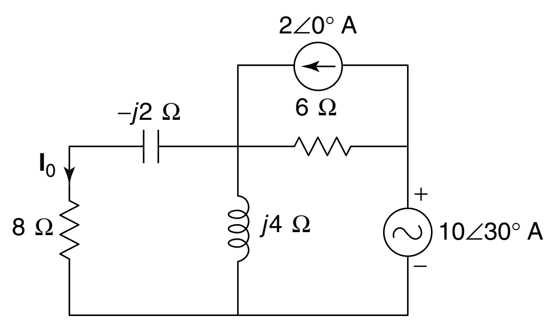

Fig. 6.35

Solution

Step I When the $10\angle 30°$ V source is acting alone (Fig. 6.36)

$$\mathbf{Z}_T = 6 + \frac{j4(8 - j2)}{j4 + 8 - j2} = 8.64\angle 24.12°\ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 30°}{8.64\angle 24.12°} = 1.16\angle 5.88° \text{ A}$$

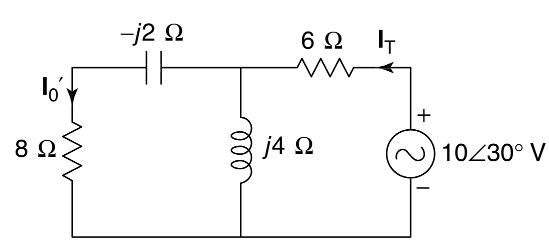

Fig. 6.36

By current division rule,

$$\mathbf{I}_0' = 1.16\angle 5.88° \times \frac{j4}{8 - j2 + j4} = 0.56\angle 81.84° \text{ A} \quad (\downarrow)$$

Step II When the $2\angle 0°$ A source is acting alone (Fig. 6.37)

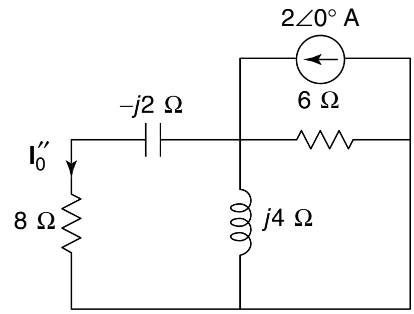

Fig. 6.37

The network can be redrawn as shown in Fig. 6.38.

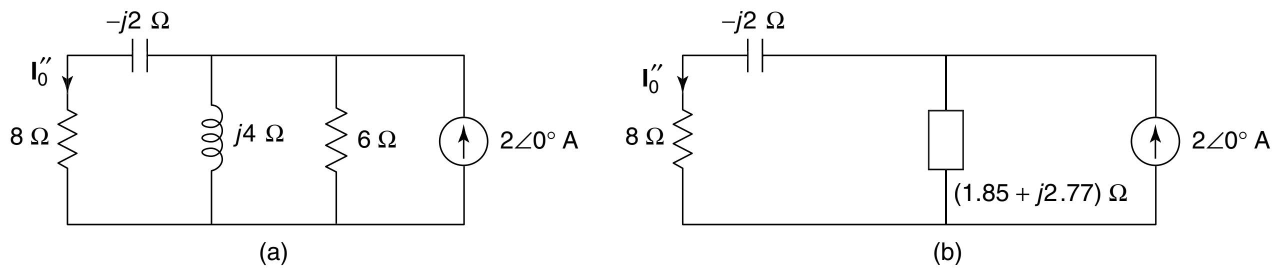

Fig. 6.38

By current division rule,

$$\mathbf{I}_0'' = 2\angle 0° \times \frac{1.85 + j2.77}{1.85 + j2.77 + 8 - j2} = 0.67\angle 51.83° \text{ A} \ (\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0'' = 0.56\angle 81.84° + 0.67\angle 51.83° = 1.19\angle 65.46° \text{ A} \ (\downarrow)$$

Example 6.23 *Find the current through the j5 Ω branch for the network shown in Fig. 6.39.*

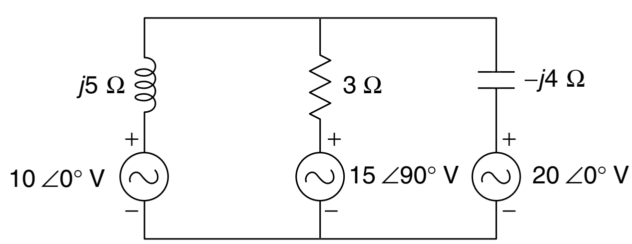

Fig. 6.39

Solution

Step I When the $10\angle 0°$ V source is acting alone (Fig. 6.40)

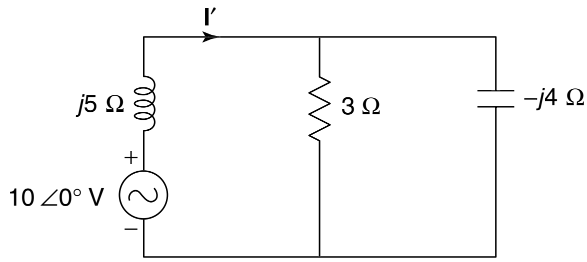

Fig. 6.40

$$\mathbf{Z}_T = j5 + \frac{3(-j4)}{3-j4} = 4.04\angle 61.66°\ \Omega$$

$$\mathbf{I}' = \frac{10\angle 0°}{4.04\angle 61.66°} = 2.48\angle -61.66°\text{A} \quad (\rightarrow)$$

Step II When the $15\angle 90°$ V source is acting alone (Fig. 6.41)

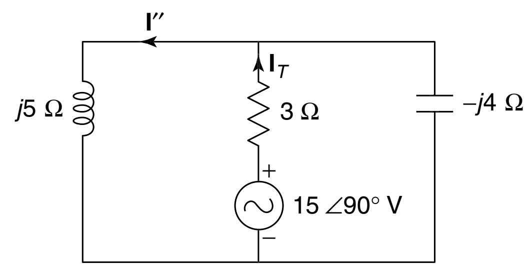

Fig. 6.41

$$\mathbf{Z}_T = 3 + \frac{(j5)(-j4)}{j5-j4} = 20.22\angle -81.47°\ \Omega$$

$$\mathbf{I}_T = \frac{15\angle 90°}{20.22\angle -81.47°} = 0.74\angle 171.47°\ \text{A}$$

By current division rule,

$$\mathbf{I}'' = 0.74\angle 171.47° \times \frac{-j4}{-j4+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A} \quad (\rightarrow)$$

Step III When the $20\ \angle 0°$ V source is acting along (Fig. 6.42)

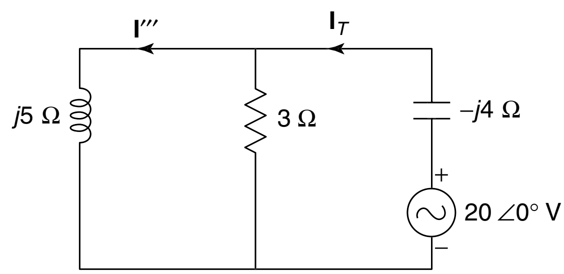

Fig. 6.42

$$\mathbf{Z}_T = -j4 + \frac{3(j5)}{3+j5} = 3.47\angle -50.51°\ \Omega$$

$$\mathbf{I}_T = \frac{20\angle 0°}{3.47\angle -50.51°} = 5.76\angle 50.51°\ \text{A}$$

By current division rule,

$$\mathbf{I}''' = 5.76\angle 50.51° \times \frac{3}{3+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A}\ (\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 2.48\angle -61.66° + 2.96\angle 171.47° + 2.96\angle 171.47° = 4.86\angle -164.41° \text{ A}$$

Example 6.24 *Find the voltage drop across the capacitor for the network shown in Fig. 6.43.*

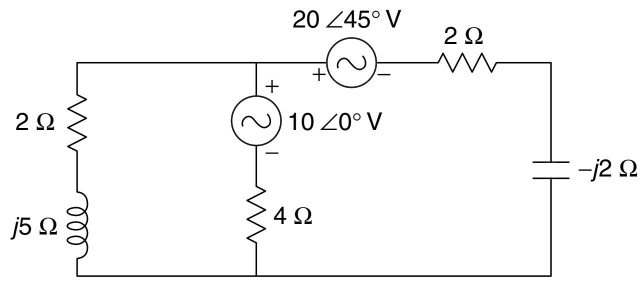

Fig. 6.43

Solution

Step I When the $10\angle 0°$ V source is acting alone (Fig. 6.44)

$$\mathbf{Z}_T = 4 + \frac{(2+j5)(2-j2)}{2+j5+2-j2}$$

$$= 7\angle -5.91° \ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 0°}{7\angle -5.91°} = 1.43\angle 5.91° \text{ A}$$

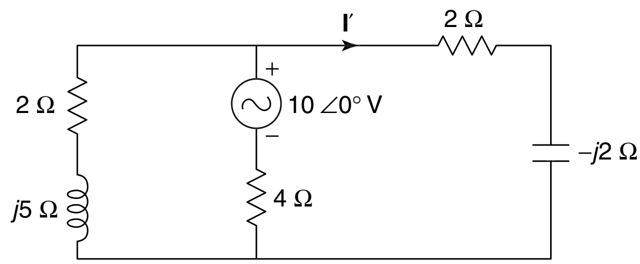

Fig. 6.44

By current division rule,

$$\mathbf{I}' = (1.43\angle 5.91°)\left(\frac{2+j5}{2+j5+2-j2}\right) = 1.54\angle 37.24° \text{ A} \quad (\rightarrow)$$

Step II When the $20\angle 45°$ V source is acting alone (Fig. 6.45)

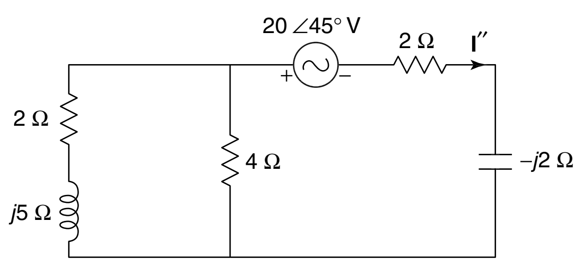

Fig. 6.45

$$\mathbf{Z}_T = (2-j2) + \frac{4(2+j5)}{4+2+j5} = 4.48\angle -8.84° \ \Omega$$

$$\mathbf{I}'' = \frac{20\angle 45°}{4.48\angle -8.84°} = 4.46\angle 53.84° \text{ A} \quad (\leftarrow) = -4.46\angle 53.84° \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 1.54\angle 37.24 - 4.46\angle 53.84° = 3.01\angle -117.78° \text{ A}$$

$$\mathbf{V}_c = (-j2)\mathbf{I} = (-j2)\ (3.01\angle -117.78°) = 6.02\angle 152.22° \text{ V}$$

Example 6.25 *Find the node voltage V_2 in the network of Fig. 6.46.*

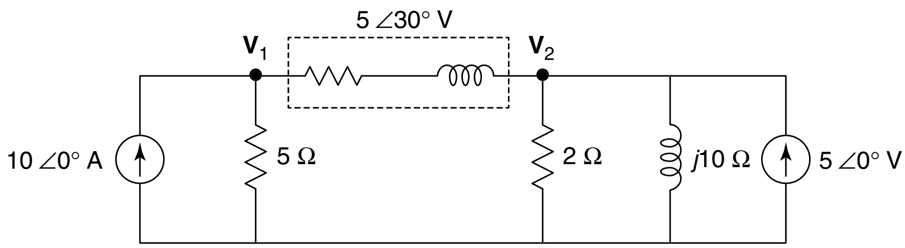

Fig. 6.46

Solution

Step I When the $10\angle 0°$ A source is acting alone (Fig. 6.47)

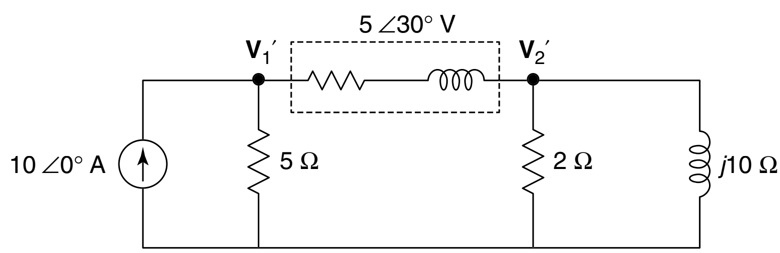

Fig. 6.47

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1'}{5} + \frac{\mathbf{V}_1' - \mathbf{V}_2'}{5\angle 30°} = 10\angle 0°$$

$$\left(\frac{1}{5} + \frac{1}{5\angle 30°}\right)\mathbf{V}_1' - \frac{1}{5\angle 30°}\mathbf{V}_2' = 10\angle 0°$$

$$(0.37 - j0.1)\mathbf{V}_1' - (0.17 - j0.1)\mathbf{V}_2' = 10\angle 0° \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2' - \mathbf{V}_1'}{5\angle 30°} + \frac{\mathbf{V}_2'}{2} + \frac{\mathbf{V}_2'}{j10} = 0$$

$$-\frac{1}{5\angle 30°}\mathbf{V}_1' + \left(\frac{1}{5\angle 30°} + \frac{1}{2} + \frac{1}{j10}\right)\mathbf{V}_2' = 0$$

$$-(0.17 - j0.1)\mathbf{V}_1' + (0.67 - j0.2)\mathbf{V}_2' = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2' = \frac{\begin{vmatrix} 0.37 - j0.1 & 10\angle 0^\circ \\ -(0.17 - j0.1) & 0 \end{vmatrix}}{\begin{vmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{vmatrix}} = 8.57\angle -3.36^\circ \text{ V}$$

Step II When the $5\angle 0^\circ$ A source is acting alone (Fig. 6.48)

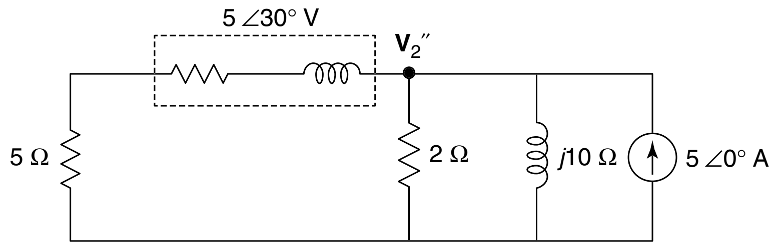

Fig. 6.48

$$\frac{\mathbf{V}_2''}{5\angle 30^\circ + 5} + \frac{\mathbf{V}_2''}{2} + \frac{\mathbf{V}_2''}{j10} = 5\angle 0^\circ$$

$$(0.61\angle -11.93^\circ)\mathbf{V}_2'' = 5\angle 0^\circ$$

$$\mathbf{V}_2'' = 8.2\angle 11.93^\circ \text{ V}$$

Step III By superposition theorem,

$$\mathbf{V}_2 = \mathbf{V}_2' + \mathbf{V}_2'' = 8.57\angle -3.36^\circ + 8.2\angle 11.93^\circ = 16.62\angle 4.12^\circ \text{ V}$$

Example 6.26 *Find current through inductor in the network of Fig. 6.49.*

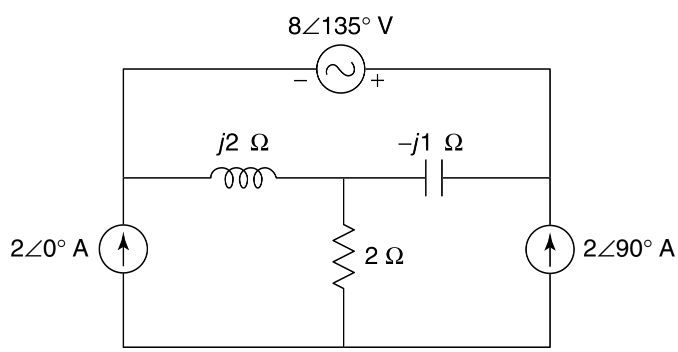

Fig. 6.49

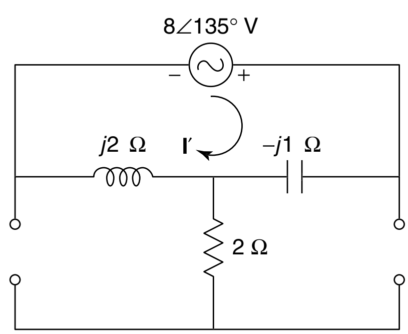

Fig. 6.50

Solution

Step I When the $8\angle 135^\circ$ V source is acting alone (Fig. 6.50)

Applying KVL to the mesh,

$$8\angle 135^\circ - (-j1)\mathbf{I}' - j2\mathbf{I}' = 0$$

$$\mathbf{I}' = \frac{8\angle 135^\circ}{j1} = 8\angle 45^\circ \text{ A } (\leftarrow) = 8\angle -135^\circ \text{ A } (\rightarrow)$$

Step II When the $2\angle 0°$ A source is acting alone (Fig. 6.51)

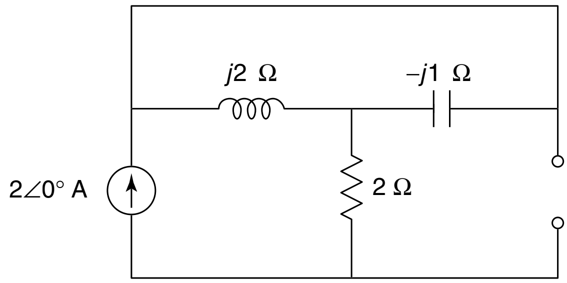

Fig. 6.51

The network can be redrawn as shown in Fig. 6.52.
By current division rule,

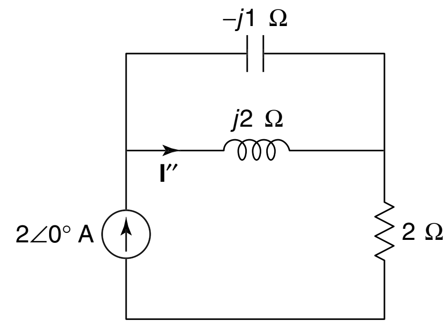

Fig. 6.52

$$\mathbf{I}'' = 2\angle 0°\left(\frac{-j1}{-j1+j2}\right) = 2\angle 0°\left(\frac{-j1}{j1}\right) = 2\angle 180° \text{ A}(\rightarrow)$$

Step III When the $2\angle 90°$ A source is acting alone (Fig. 6.53)

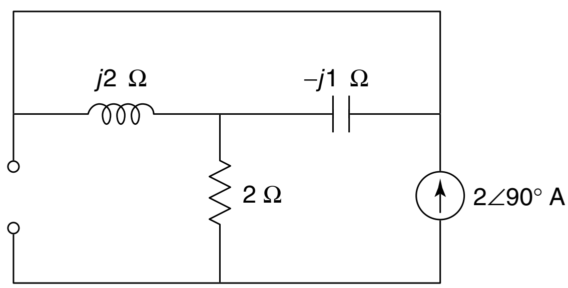

Fig. 6.53

The network can be redrawn as shown in Fig. 6.54.
By current division rule,

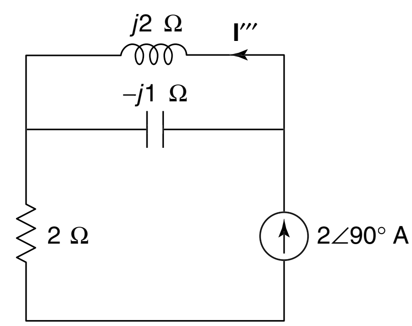

Fig. 6.54

$$\mathbf{I}''' = 2\angle 90°\left(\frac{-j1}{-j1+j2}\right) = 2\angle -90° \text{ A } (\leftarrow) = 2\angle 90° \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 8\angle -135° + 2\angle 180° + 2\angle 90° = 8.49\angle -154.47° \text{A}$$

Example 6.27 *Determine the source voltage V_S so that the current through 2 Ω resistor is zero in the network of Fig. 6.55.*

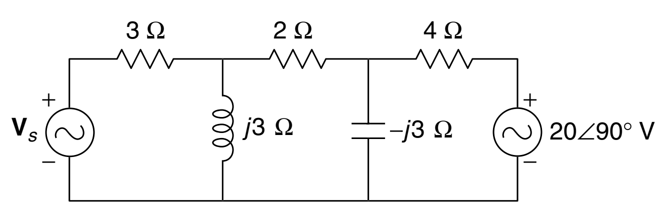

Fig. 6.55

Solution

Step I When the voltage source $\mathbf{V}_s$ is acting alone (Fig. 6.56)

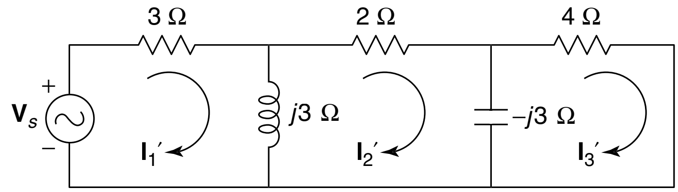

Fig. 6.56

Appling KVL to Mesh 1,

$$\mathbf{V}_s - 3\mathbf{I}_1' - j3(\mathbf{I}_1' - \mathbf{I}_2') = 0$$

$$(3 + j3)\mathbf{I}_1' - j3\mathbf{I}_2' = \mathbf{V}_s \qquad \text{...(i)}$$

Appling KVL to Mesh 2,

$$-j3(\mathbf{I}_2' - \mathbf{I}_1') - 2\mathbf{I}_2' + j3(\mathbf{I}_2' - \mathbf{I}_3') = 0$$

$$-j3\mathbf{I}_1' + 2\mathbf{I}_2' + j3\mathbf{I}_3' = 0 \qquad \text{...(ii)}$$

Appling KVL to Mesh 3,

$$-j3(\mathbf{I}_3' - \mathbf{I}_2') - 4\mathbf{I}_3' = 0$$

$$j3\mathbf{I}_2' + (4 - j3)\mathbf{I}_3' = 0 \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1' \\ \mathbf{I}_2' \\ \mathbf{I}_3' \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2' = \frac{\begin{vmatrix} 3+j3 & \mathbf{V}_s & 0 \\ -j3 & 0 & j3 \\ 0 & 0 & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{(9 + j12)\mathbf{V}_s}{\Delta}$$

Step II When the 20 ∠90° V source is acting alone (Fig. 6.57)

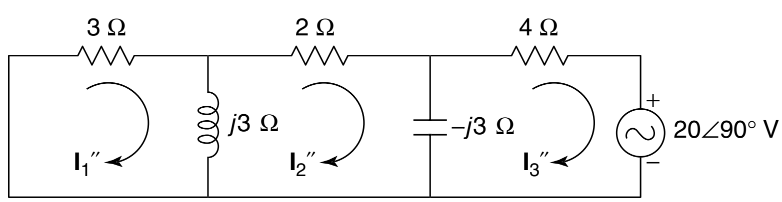

Fig. 6.57

Applying KVL to Mesh 1,

$$-3\mathbf{I}_1'' - j3(\mathbf{I}_1'' - \mathbf{I}_2'') = 0$$

$$(3 + j3)\mathbf{I}_1'' - j3\mathbf{I}_2'' = 0 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j3(\mathbf{I}_2'' - \mathbf{I}_1'') - 2\mathbf{I}_2'' + j3(\mathbf{I}_2'' - \mathbf{I}_3'') = 0$$

$$-j3\mathbf{I}_1'' + 2\mathbf{I}_2'' + j3\mathbf{I}_3'' = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$j3(\mathbf{I}_3'' - \mathbf{I}_2'') - 4\mathbf{I}_3'' - 20\angle 90° = 0$$

$$j3\,\mathbf{I}_2'' + (4 - j3)\,\mathbf{I}_3'' = -20\angle 90° \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1'' \\ \mathbf{I}_2'' \\ \mathbf{I}_3'' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20\angle 90° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2'' = \frac{\begin{vmatrix} 3+j3 & 0 & 0 \\ -j3 & 0 & j3 \\ 0 & -20\angle 90° & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{-180 - j180}{\Delta}$$

Step III By superposition theorem,

$$\mathbf{I}_2 = \mathbf{I}_2' + \mathbf{I}_2'' = \frac{(9 + j12)\mathbf{V}_s + (-180 - j180)}{\Delta} = 0$$

$$(9 + j12)\mathbf{V}_s + (-180 - j180) = 0$$

$$(9 + j12)\mathbf{V}_s = 180 + j180$$

$$\mathbf{V}_s = 16.97\angle -8.13° \text{ V}$$

6.5 THEVENIN'S THEOREM

Thevenin's theorem gives us a method for simplifying a network. In Thevenin's theorem, *any linear network can be replaced by a voltage source* $\mathbf{V}_{\text{Th}}$ *in series with an impedance* $\mathbf{Z}_{\text{Th}}$.

Example 6.28 *Obtain Thevenin's equivalent network for the terminals A and B in Fig. 6.58.*

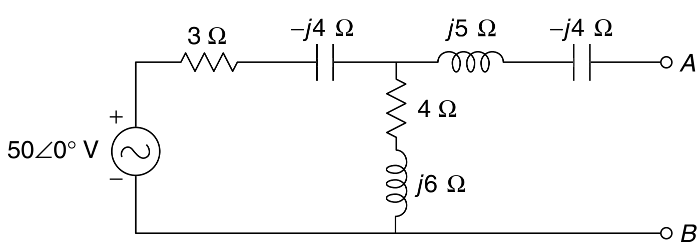

Fig. 6.58

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.59)

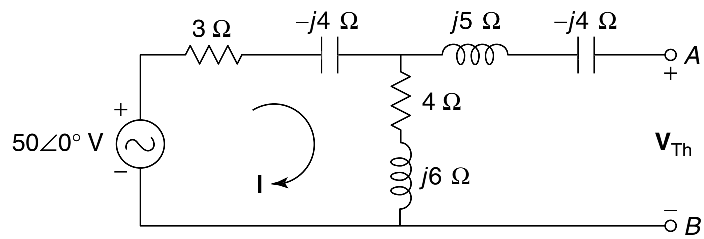

Fig. 6.59

Applying KVL to the mesh,

$$50\angle 0° - (3 - j4)\,\mathbf{I} - (4 + j6)\,\mathbf{I} = 0$$

$$\mathbf{I} = \frac{50\angle 0°}{(3-j4)+(4+j6)} = 6.87\angle -15.95° \text{ A}$$

$$\mathbf{V}_{\text{Th}} = (4 + j6)\,\mathbf{I} = (4 + j6)\,(6.87\angle -15.95°) = 49.5\angle 40.35° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.60)

$$\mathbf{Z}_{\text{Th}} = (j5 - j4) + \frac{(3-j4)(4+j6)}{(3-j4)+(4+j6)} = 4.83\angle -1.13°\ \Omega$$

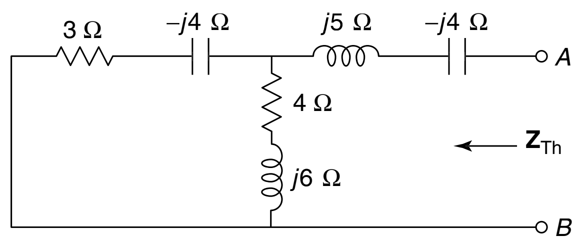

Fig. 6.60

Step III Thevenin's Equivalent Network (Fig. 6.61)

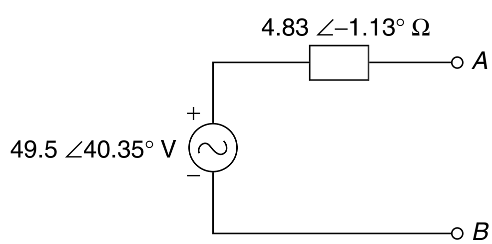

Fig. 6.61

Example 6.29 *Find Thevenin's equivalent network for Fig. 6.62.*

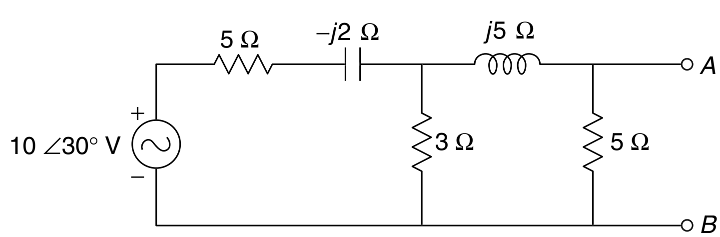

Fig. 6.62

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.63)

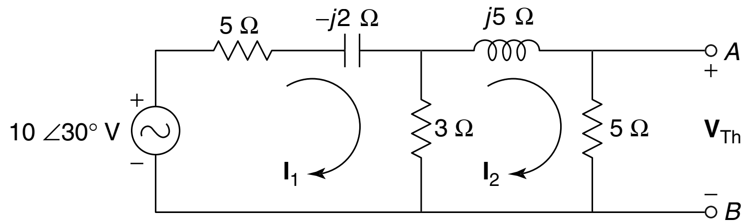

Fig. 6.63

Applying KVL to Mesh 1,

$$10\angle 30^\circ - (5 - j2)\,\mathbf{I}_1 - 3(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(8 - j2)\,\mathbf{I}_1 - 3\mathbf{I}_2 = 10\angle 30^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-3(\mathbf{I}_2 - \mathbf{I}_1) - j5\,\mathbf{I}_2 - 5\,\mathbf{I}_2 = 0$$

$$-3\mathbf{I}_1 + (8 + j5)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form;

$$\begin{bmatrix} 8 - j2 & -3 \\ -3 & 8 + j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 8 - j2 & 10\angle 30^\circ \\ -3 & 0 \end{vmatrix}}{\begin{vmatrix} 8 - j2 & -3 \\ -3 & 8 + j5 \end{vmatrix}} = 0.433\angle 9.7^\circ \text{ A}$$

$$\mathbf{V}_{Th} = 5\mathbf{I}_2 = 5(0.433\angle 9.7^\circ) = 2.16\angle 9.7^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.64)

$$\mathbf{Z}_{Th} = \left[\left\{\frac{(5 - j2)3}{5 - j2 + 3}\right\} + j5\right] \| \, 5$$

$$= [1.94 - j0.265 + j5] \| \, 5 = (1.94 + j4.735) \| \, 5$$

$$= \frac{(1.94 + j4.735)5}{6.94 + j4.735} = 3.04\angle 33.4^\circ \, \Omega$$

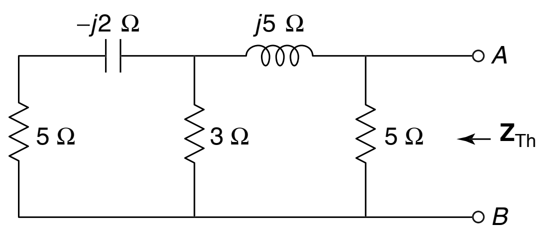

Fig. 6.64

Step III Thevenin's equivalent Network (Fig. 6.65)

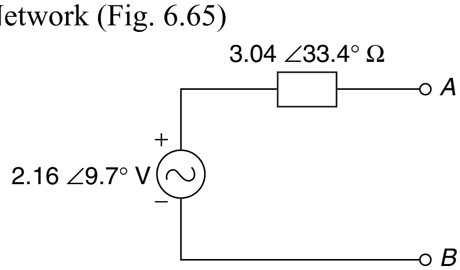

Fig. 6.65

Example 6.30 *Obtain Thevenin's equivalent network for Fig. 6.66.*

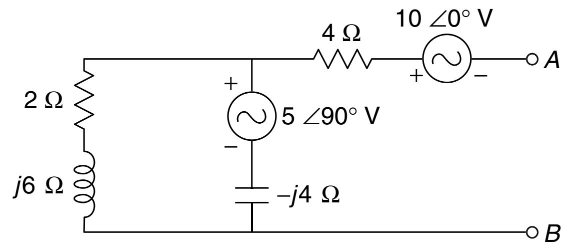

Fig. 6.66

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$

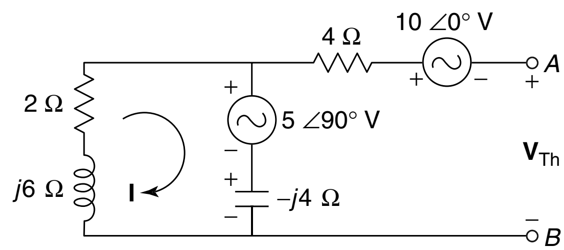

Fig. 6.67

Applying KVL to the mesh,

$$(2 + j6 - j4)\mathbf{I} - 5\angle 90° = 0$$

$$\mathbf{I} = \frac{5\angle 90°}{2 + j2} = 1.77\angle 45° \text{ A}$$

$$\mathbf{V}_{\text{Th}} = (-j4)\,\mathbf{I} + 5\angle 90° - 10\angle 0° = (4\angle{-90°})(1.77\angle 45°) + 5\angle 90° - 10\angle 0° = 18\angle 146.31° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.67)

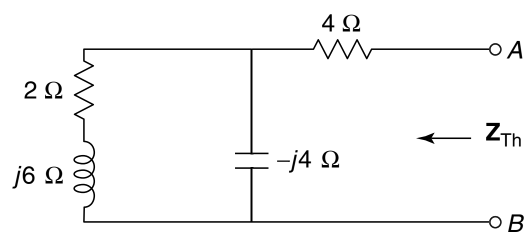

Fig. 6.68

$$\mathbf{Z}_{\text{Th}} = 4 + \frac{(2 + j6)(-j4)}{2 + j2} = 11.3\angle{-44.93°}\ \Omega$$

Step III Thevenin's Equivalent Network

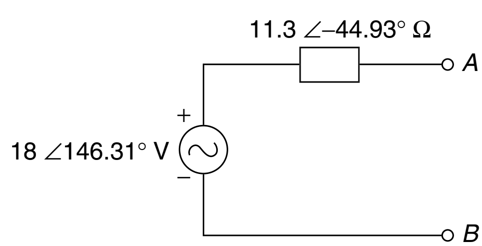

Fig. 6.69

Example 6.31 *Obtain Thevenin's equivalent network for Fig. 6.70.*

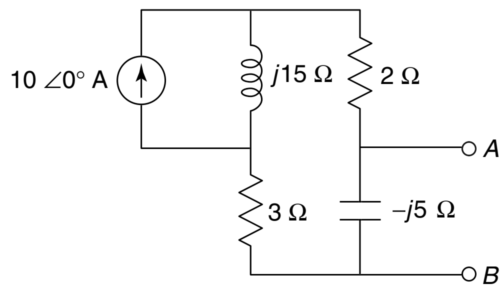

Fig. 6.70

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.71)

By current division rule,

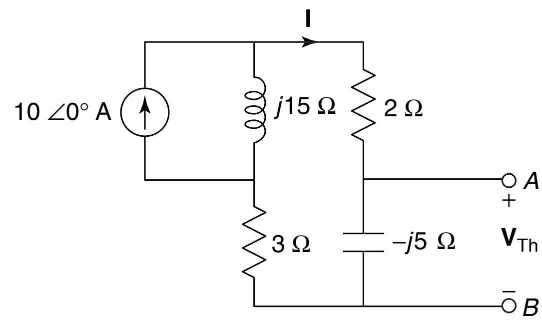

Fig. 6.71

$$\mathbf{I} = \frac{(10\angle 0°)(j15)}{5 - j5 + j15} = 13.42\angle 26.57° \text{ A}$$

$$\mathbf{V}_{\text{Th}} = (-j5)\,\mathbf{I}$$
$$= (5\angle -90°)\,(13.42\angle 26.57°) = 67.08\angle -63.43° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.72)

$$\mathbf{Z}_{\text{Th}} = \frac{(-j5)(5 + j15)}{-j5 + 5 + j15} = 7.07\angle -81.86° \ \Omega$$

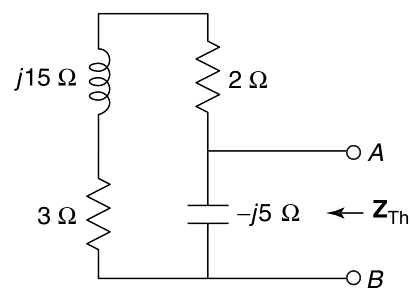

Fig. 6.72

Step III Thevenin's Equivalent Network

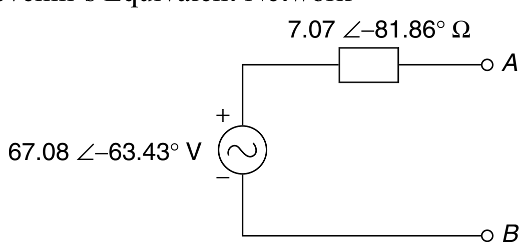

Fig. 6.73

Example 6.32 *Obtain Thevenin's equivalent network for Fig. 6.74.*

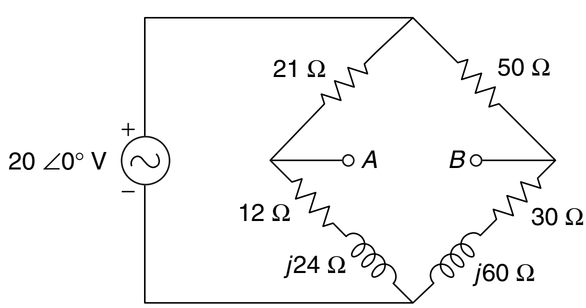

Fig. 6.74

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig 6.75)

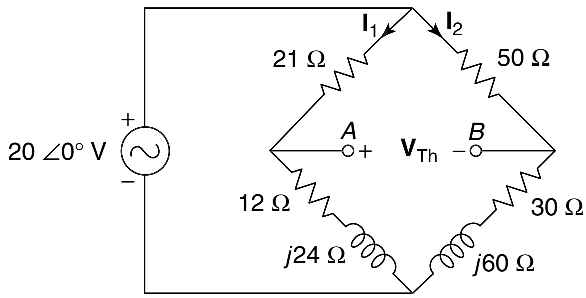

Fig. 6.75

$$\mathbf{I}_1 = \frac{20\angle 0^\circ}{21+12+j24} = 0.49\angle -36.02^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{20\angle 0^\circ}{80+j60} = 0.2\angle -36.86^\circ \text{ A}$$

$$\begin{aligned}\mathbf{V}_{Th} &= (12+j24)\,\mathbf{I}_1 - (30+j60)\,\mathbf{I}_2 \\ &= (26.83\ \angle 63.43^\circ)\,(0.49\ \angle -36.02^\circ) - (67.08\ \angle 63.43^\circ)\,(0.2\ \angle -36.86^\circ) \\ &= 0.33\ \angle 171.12^\circ \text{ V}\end{aligned}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.76)

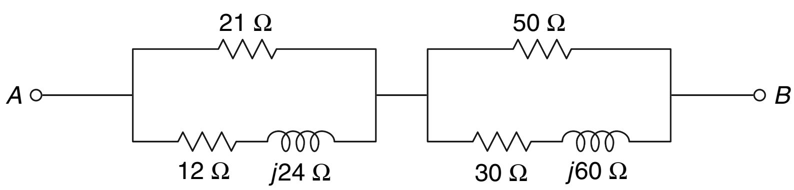

Fig. 6.76

$$\mathbf{Z}_{Th} = \frac{21(12+j24)}{33+j24} + \frac{50(30+j60)}{80+j60} = 47.4\angle 26.8^\circ\ \Omega$$

Step III Thevenin's Equivalent Network

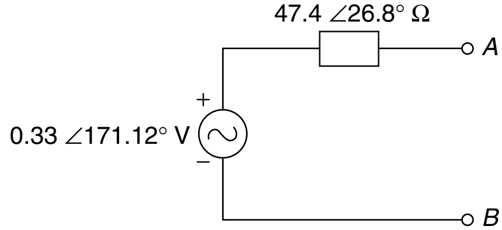

Fig. 6.77

Example 6.33 *Find Thevenin's equivalent network across terminals A and B for Fig. 6.78.*

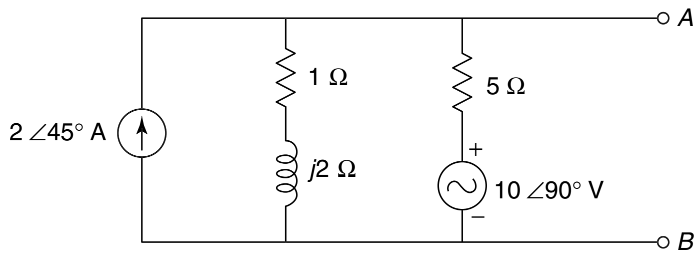

Fig. 6.78

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.79)

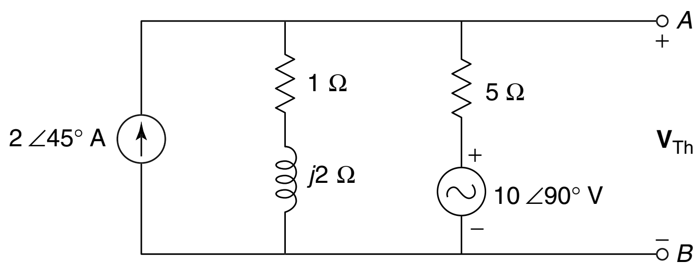

Fig. 6.79

Applying KCL at the node,

$$\frac{\mathbf{V}_{Th}}{1+j2}+\frac{\mathbf{V}_{Th}-10\angle 90^\circ}{5}=2\angle 45^\circ$$

$$\left(\frac{1}{1+j2}+\frac{1}{5}\right)\mathbf{V}_{Th}=2\angle 45^\circ+2\angle 90^\circ$$

$$(0.57\angle -45^\circ)\mathbf{V}_{Th}=3.7\angle 67.5^\circ$$

$$\mathbf{V}_{Th}=6.49\angle 112.5^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.80)

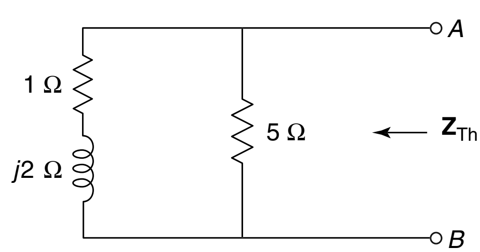

Fig. 6.80

$$\mathbf{Z}_{Th}=\frac{5(1+j2)}{5+1+j2}=1.77\angle 45^\circ\ \Omega$$

Step III Thevenin's Equivalent Network (Fig. 6.81)

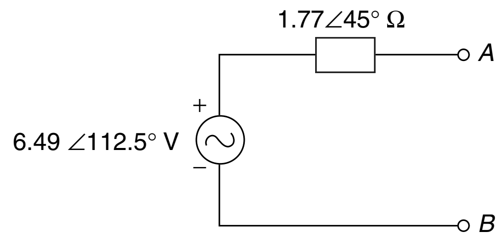

Fig. 6.81

Example 6.34 *Find the current through the* $(5+j2)\ \Omega$ *impedance in the network of Fig. 6.82.*

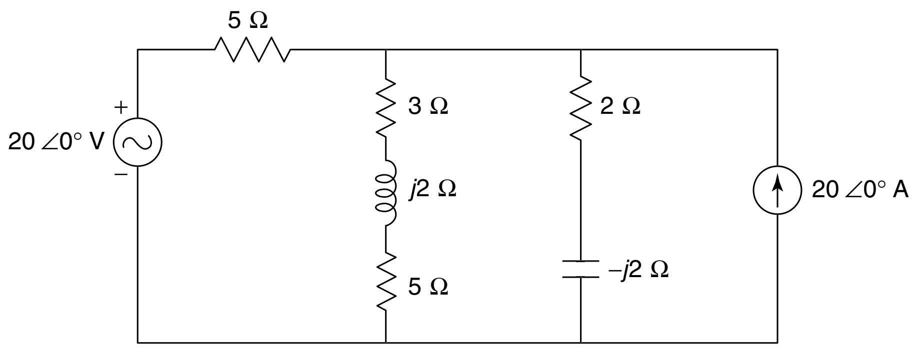

Fig. 6.82

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.83)

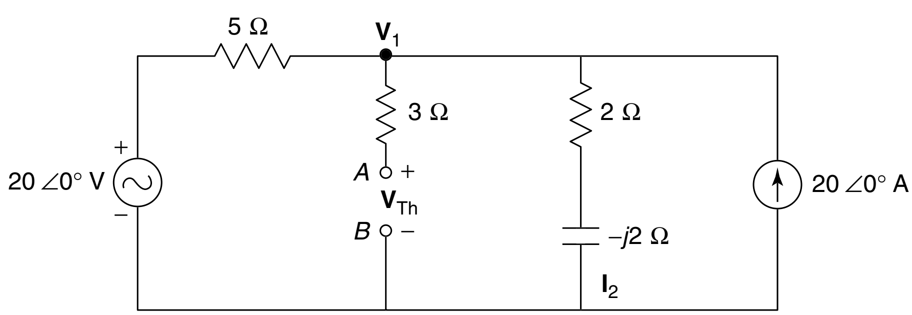

Fig. 6.83

Applying KCL at the node,

$$\frac{\mathbf{V}_1 - 20\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{2-j2} = 20\angle 0^\circ$$

$$\left(\frac{1}{5} + \frac{1}{2-j2}\right)\mathbf{V}_1 = 20\angle 0^\circ + 4\angle 0^\circ$$

$$0.51\angle 29.05^\circ\ \mathbf{V}_1 = 24\angle 0^\circ$$

$$\mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

$$\mathbf{V}_{Th} = \mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\mathrm{Th}}$ (Fig. 6.84)

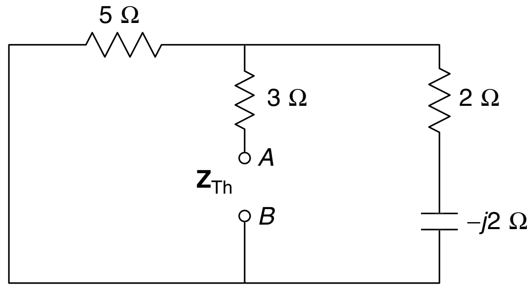

Fig. 6.84

$$\mathbf{Z}_{\mathrm{Th}} = 3 + \frac{5(2 - j2)}{5 + 2 - j2} = 4.79\angle{-11.35°}\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.85)

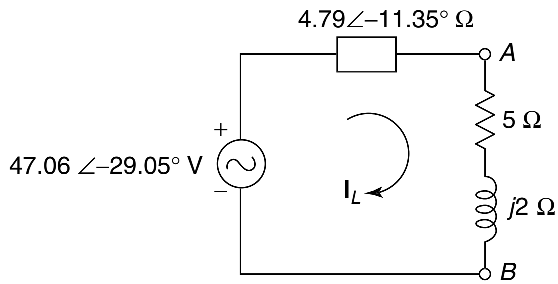

Fig. 6.85

$$\mathbf{I}_L = \frac{47.06\angle{-29.05°}}{4.79\angle{-11.35°} + 5 + j2} = 4.73\angle{-39.96°}\ \text{A}$$

Example 6.35 *Find the current through the 5 Ω resistor in the network of Fig. 6.86.*

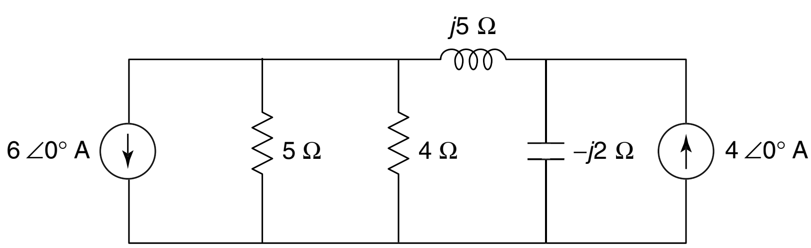

Fig. 6.86

Solution

Step I Calculation of $\mathbf{V}_{\mathrm{Th}}$ (Fig. 6.87)

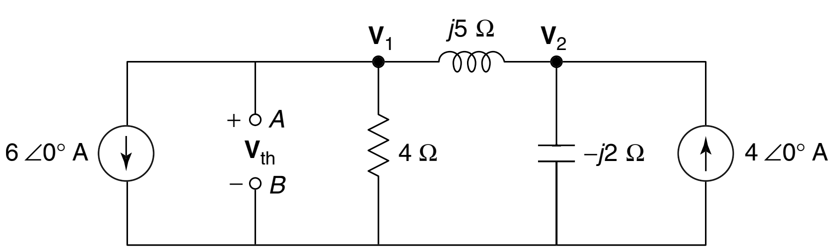

Fig. 6.87

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1}{4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j5} + 6\angle 0^\circ = 0$$

$$\left(\frac{1}{4} + \frac{1}{j5}\right)\mathbf{V}_1 - \frac{1}{j5}\mathbf{V}_2 = -6\angle 0^\circ$$

$$(0.25 - j0.2)\mathbf{V}_1 + j0.2\mathbf{V}_2 = -6\angle 0^\circ \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j5} + \frac{\mathbf{V}_2}{-j2} = 4\angle 0^\circ$$

$$\left(-\frac{1}{j5}\right)\mathbf{V}_1 + \left(\frac{1}{j5} - \frac{1}{j2}\right)\mathbf{V}_2 = 4\angle 0^\circ$$

$$j0.2\,\mathbf{V}_1 + j0.3\,\mathbf{V}_2 = 4\angle 0^\circ \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.25 - j0.2 & j0.2 \\ j0.2 & j0.3 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} -6\angle 0^\circ \\ 4\angle 0^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} -6\angle 0^\circ & j0.2 \\ 4\angle 0^\circ & j0.3 \end{vmatrix}}{\begin{vmatrix} 0.25 - j0.2 & j0.2 \\ j0.2 & j0.3 \end{vmatrix}} = 20.8\angle -126.87^\circ \text{ V}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_1 = 20.8\angle -126.87^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.88)

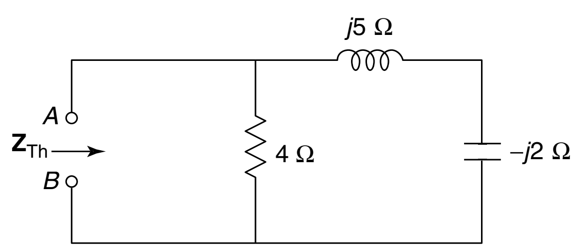

Fig. 6.88

$$\mathbf{Z}_{\text{Th}} = \frac{4(-j2 + j5)}{4 - j2 + j5)} = 2.4\angle 53.13^\circ \ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.89)

$$\mathbf{I}_L = \frac{20.8\angle -126.87^\circ}{2.4\angle 53.13^\circ + 5} = 3.1\angle -143.47^\circ \text{ A}$$

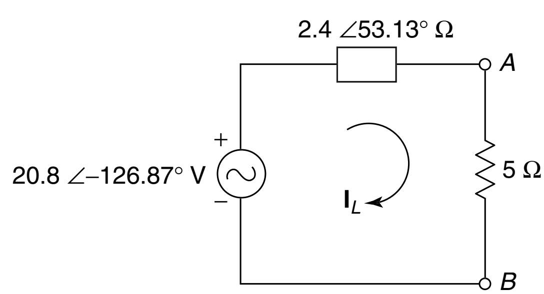

Fig. 6.89

Example 6.36 *In the network of Fig. 6.90, find the current through the 10 Ω resistor.*

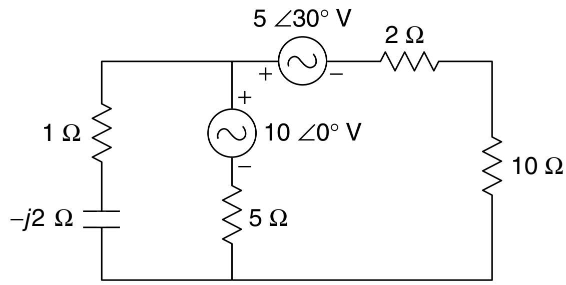

Fig. 6.90

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.91)

Applying KVL to the mesh,

$$j2\mathbf{I} - 1\mathbf{I} - 10\angle 0° - 5\mathbf{I} = 0$$

$$(j2-6)\mathbf{I} = 10\angle 0°$$

$$\mathbf{I} = 1.58\angle -161.57° \text{ A}$$

Writing $\mathbf{V}_{Th}$ equation,

$$5\mathbf{I} + 10\angle 0° - 5\angle 30° - 0 - \mathbf{V}_{Th} = 0$$

$$5(1.58\angle -161.57°) - 10\angle 0° - 5\angle 30° - \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = 5.32\angle -110.06° \text{ V}$$

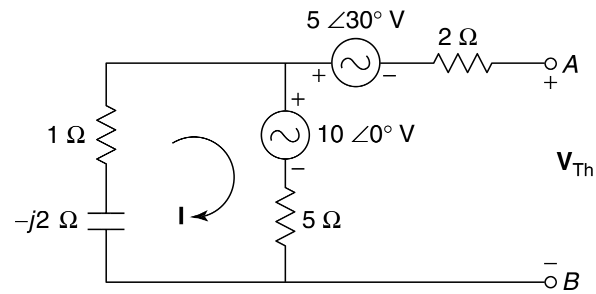

Fig. 6.91

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.92)

$$\mathbf{Z}_{Th} = 2 + \frac{5(1-j2)}{5+1-j2} = 3.48\angle -21.04° \ \Omega$$

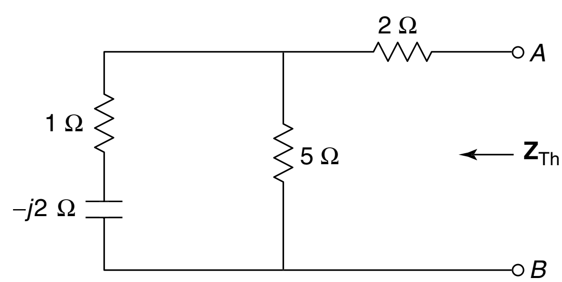

Fig. 6.92

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.93)

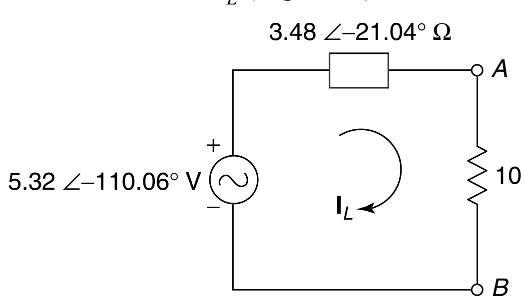

Fig. 6.93

$$\mathbf{I}_L = \frac{5.32\angle -110.06°}{3.48\angle -21.04° + 10} = 0.4\angle -104.67° \text{ A}$$

Example 6.37 *Find the current through $(4 + j6)\ \Omega$ impedance in the network of Fig. 6.94.*

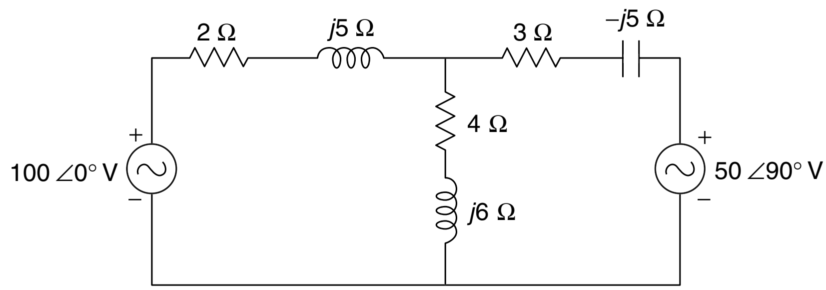

Fig. 6.94

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.95)

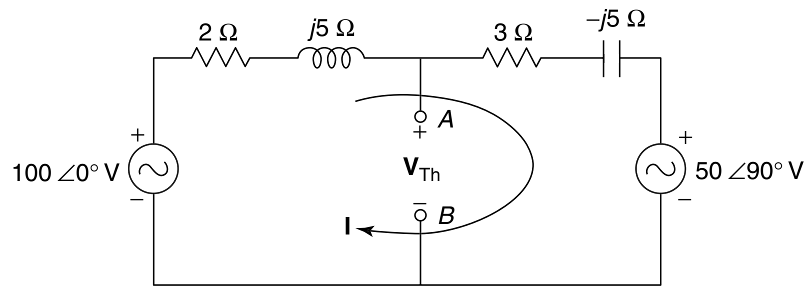

Fig. 6.95

Applying KVL to the mesh,

$$100\angle 0° - 2\mathbf{I} - j5\mathbf{I} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90° = 0$$

$$\mathbf{I} = 22.36\angle -26.57° \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$\mathbf{V}_{\text{Th}} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90° = 0$$

$$\mathbf{V}_{\text{Th}} - (3 - j5)(22.36\angle -26.57°) - 50\angle 90° = 0$$

$$\mathbf{V}_{\text{Th}} = 80.61\angle -82.88° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.96)

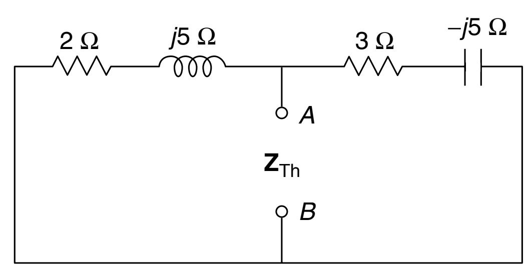

Fig. 6.96

$$\mathbf{Z}_{\text{Th}} = \frac{(2 + j5)(3 - j5)}{2 + j5 + 3 - j5} = 6.28\angle 9.16°\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.97)

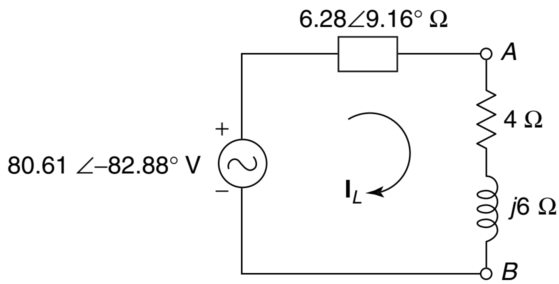

Fig. 6.97

$$\mathbf{I}_L = \frac{80.61\angle -82.88°}{6.28\angle 9.16° + 4 + j6} = 6.52\angle -117.34° \text{ A}$$

Example 6.38 *Obtain Thevenin's equivalent network across terminals A and B in Fig. 6.98.*

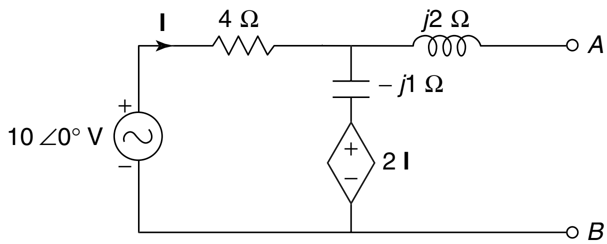

Fig. 6.98

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.99)

Applying KVL to the mesh,

$$10\angle 0° - 4\mathbf{I} + j1\mathbf{I} - 2\mathbf{I} = 0$$

$$\mathbf{I} = 1.64\angle 9.46° \text{ A}$$

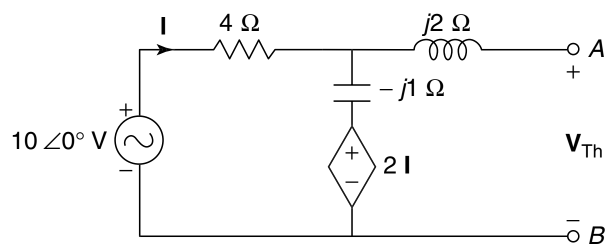

Fig. 6.99

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0° - 4\mathbf{I} - 0 - \mathbf{V}_{\text{Th}} = 0$$

$$10\angle 0° - 4(1.64\angle 9.46°) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 3.69\angle -17° \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.100)

From Fig. 6.100,

$$\mathbf{I} = \mathbf{I}_1$$

Applying KVL to Mesh 1,

$$10\angle 0° - 4\mathbf{I}_1 + j1(\mathbf{I}_1 - \mathbf{I}_2) - 2\mathbf{I} = 0$$

$$10\angle 0° - 4\mathbf{I}_1 + j1\mathbf{I}_1 - j1\mathbf{I}_2 - 2\mathbf{I}_1 = 0$$

$$(6 - j1)\mathbf{I}_1 + j1\mathbf{I}_2 = 10\angle 0° \quad \ldots\text{(i)}$$

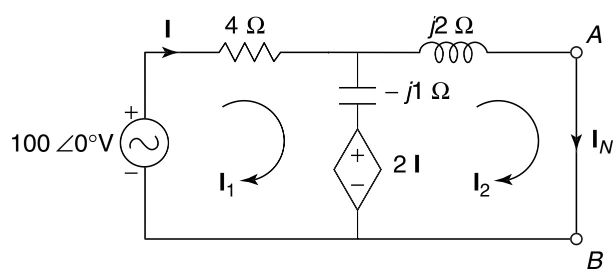

Fig. 6.100

Applying KVL to Mesh 2,

$$2\mathbf{I} + j1(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 = 0$$

$$2\mathbf{I}_1 + j1\mathbf{I}_2 - j1\mathbf{I}_1 - j_2\mathbf{I}_2 = 0$$

$$(2-j1)\mathbf{I}_1 - j1\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 6-j1 & 10\angle 0° \\ 2-j1 & 0 \end{vmatrix}}{\begin{vmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{vmatrix}} = 2.71\angle -102.53° \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 2.71\angle -102.53° \text{ A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.69\angle -17°}{2.71\angle -102.53°} = 1.36\angle 85.53° \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 6.101)

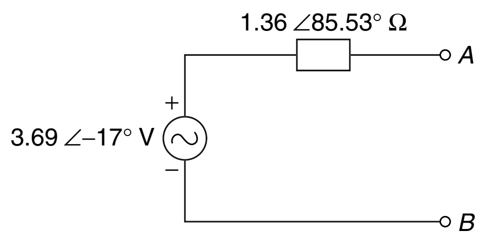

Fig. 6.101

Example 6.39 *Find Thevenin's equivalent network across terminals A and B for Fig. 6.102.*

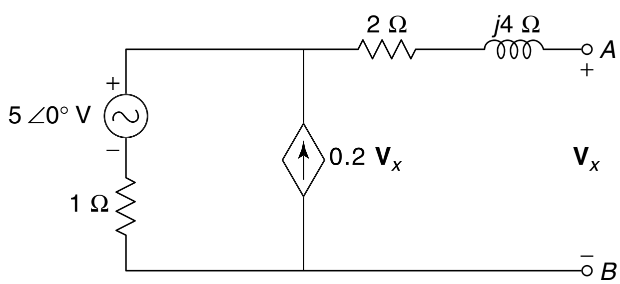

Fig. 6.102

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.103)

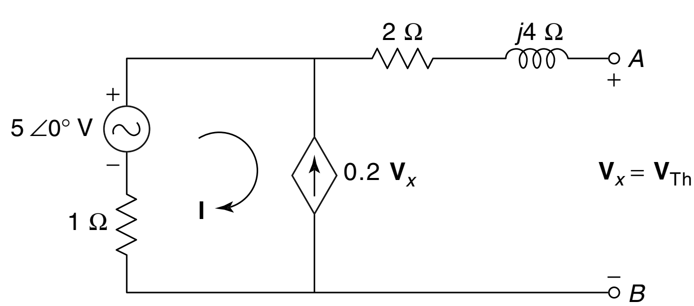

Fig. 6.103

From Fig. 6.103,

$$\mathbf{I} = -0.2\,\mathbf{V}_x \qquad \text{... (i)}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$-\mathbf{I} + 5\angle 0° - 0 - \mathbf{V}_x = 0$$

$$0.2\mathbf{V}_x + 5\angle 0° - \mathbf{V}_x = 0$$
$$\mathbf{V}_x = 6.25\angle 0° \text{ V}$$
$$\mathbf{V}_{\text{Th}} = \mathbf{V}_x = 6.25\angle 0° \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.104)

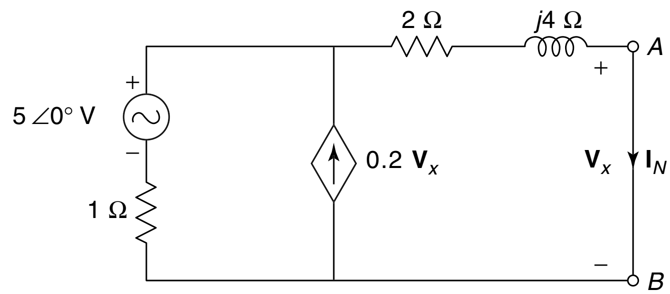

Fig. 6.104

From Fig. 6.104,

$$\mathbf{V}_x = 0$$

The dependent source depends on the controlling variable $\mathbf{V}_x$. When $\mathbf{V}_x = 0$, the dependent source vanishes, i.e. $0.2\mathbf{V}_x = 0$ as shown in Fig. 6.105.

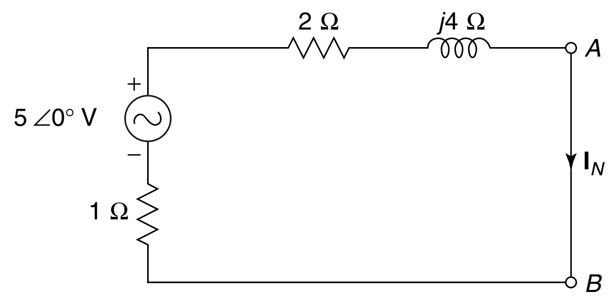

Fig. 6.105

$$\mathbf{I}_N = \frac{5\angle 0°}{1+2+j4} = 1\angle -53.13° \text{ A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{6.25\angle 0°}{1\angle -53.13°} = 6.25\angle 53.13° \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 6.106)

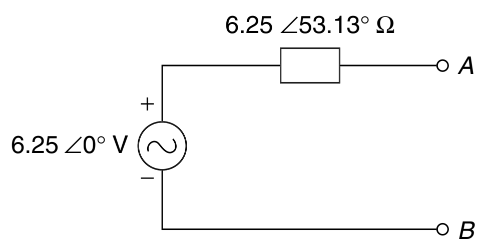

Fig. 6.106

6.6 NORTON'S THEOREM

Norton's theorem states that *any linear network can be replaced by a current source* $\mathbf{I}_N$ *parallel with an impedance* $\mathbf{Z}_N$ *where* $\mathbf{I}_N$ *is the current flowing through the short-circuited path placed across the terminals.*

Example 6.40 *Obtain Norton's equivalent network between terminals A and B as shown in Fig. 6.107.*

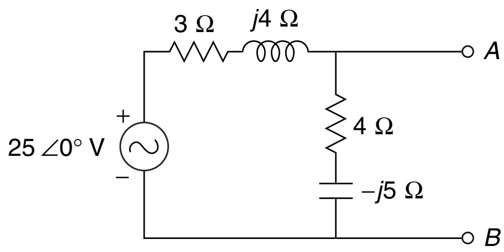

Fig. 6.107

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.108)

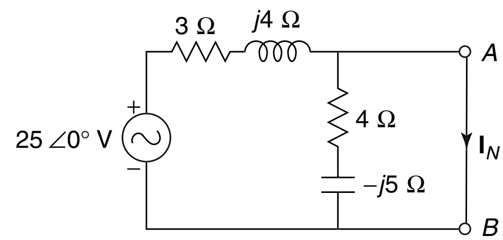

Fig. 6.108

When a short circuit is placed across $(4 - j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 6.109.

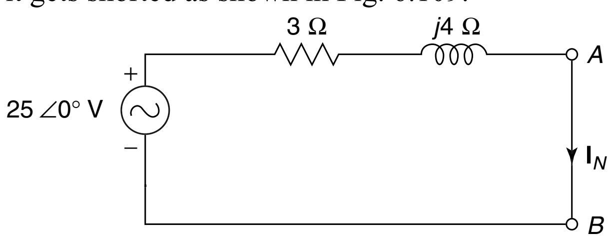

Fig. 6.109

$$\mathbf{I}_N = \frac{25\angle 0^\circ}{3 + j4} = 5\angle -53.13^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.110)

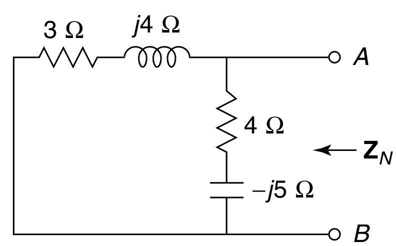

Fig. 6.110

$$\mathbf{Z}_N = \frac{(3 + j4)(4 - j5)}{3 + j4 + 4 - j5} = 4.53\angle 9.92^\circ\ \Omega$$

Step III Norton's Equivalent Network

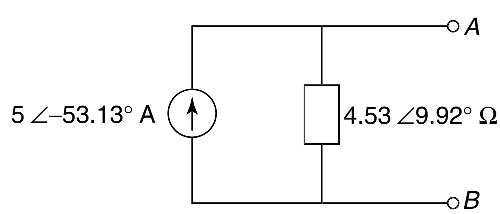

Fig. 6.111

Example 6.41 *Obtain Norton's equivalent network at the terminals A and B in Fig. 6.112.*

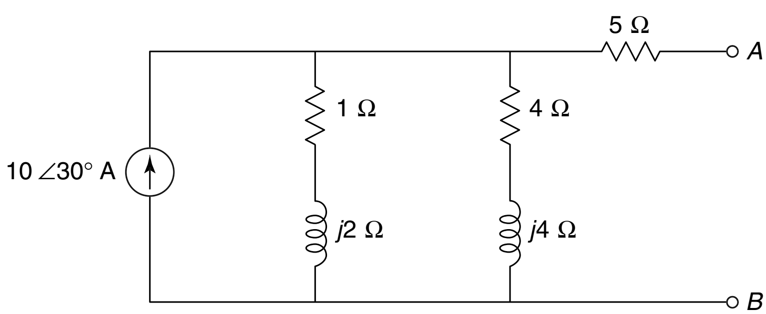

Fig. 6.112

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.113)

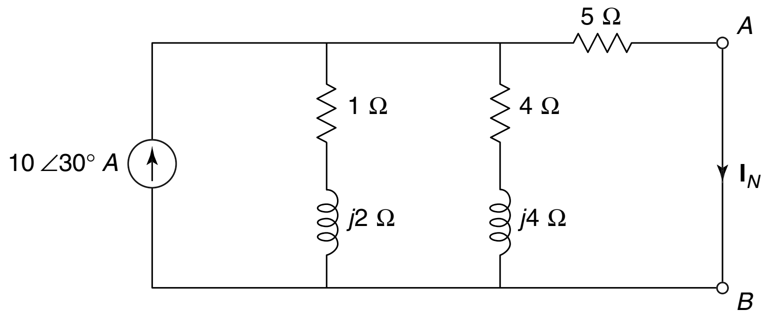

Fig. 6.113

By series-parallel reduction technique (Fig. 6.114)

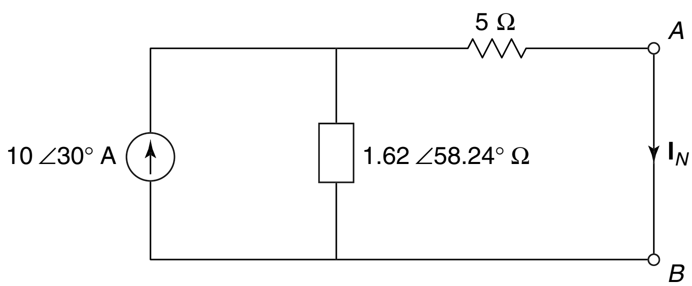

Fig. 6.114

$$\mathbf{I}_N = (10\angle 30°)\left(\frac{1.62\angle 58.24°}{1.62\angle 58.24° + 5}\right) = 2.69\angle 75° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.115)

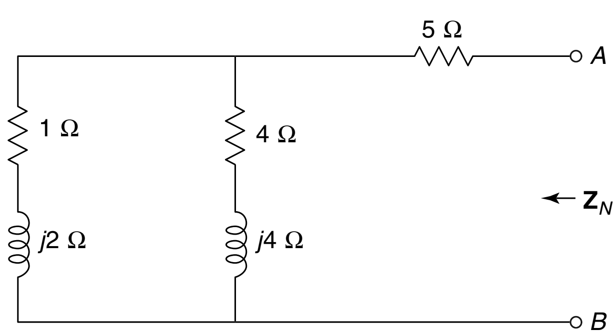

Fig. 6.115

$$\mathbf{Z}_N = 5 + \frac{(1+j2)(4+j4)}{1+j2+4+j4} = 6.01\angle 13.24° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.116)

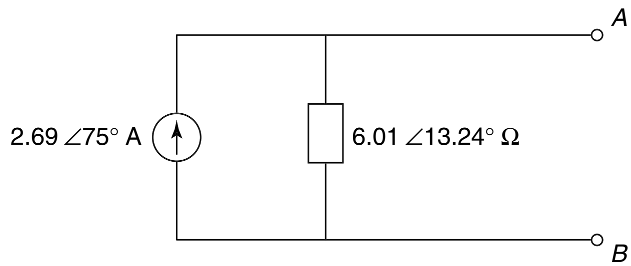

Fig. 6.116

Example 6.42 *Find Norton's equivalent network across terminals A and B in Fig. 6.117.*

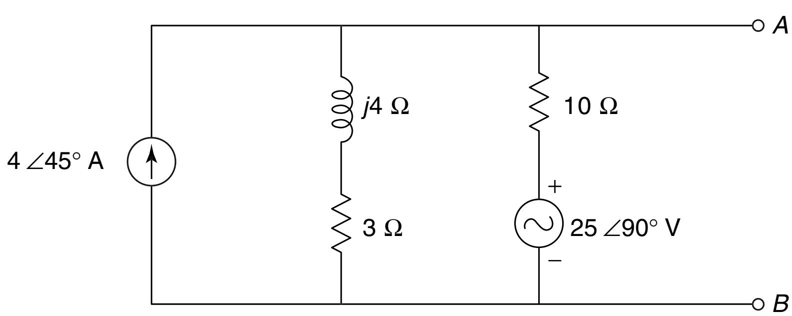

Fig. 6.117

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.118)

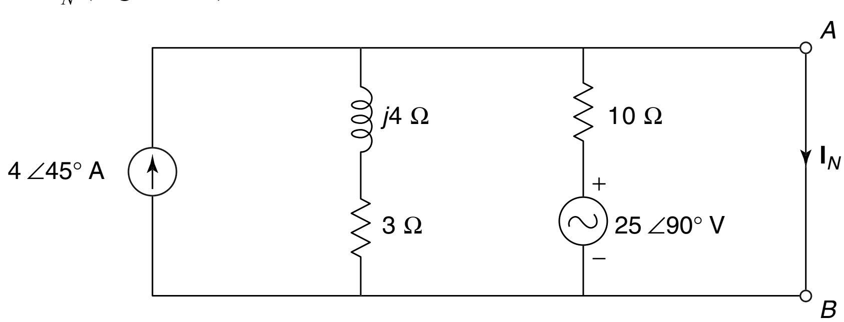

Fig. 6.118

When a short circuit is placed across the $(3+j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 6.119.

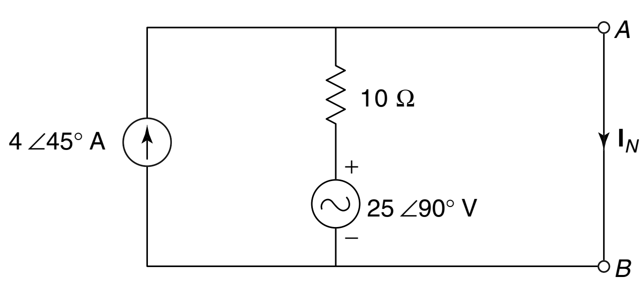

Fig. 6.119

By source transformation, the network is redrawn as shown in Fig. 6.120.

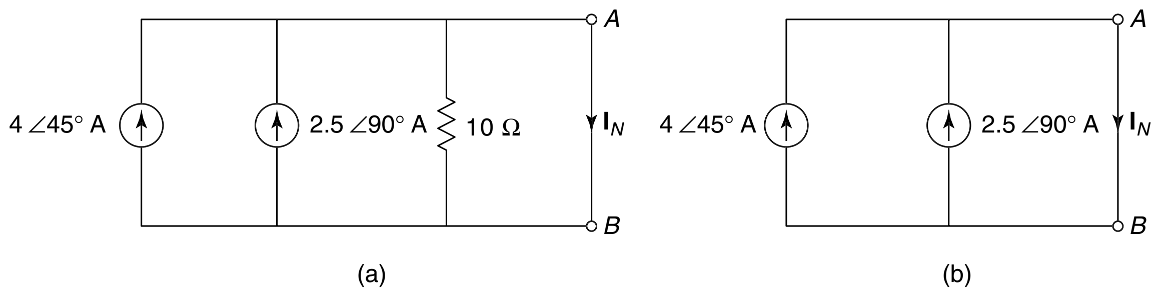

Fig. 6.120

$$I_N = 4\angle 45° + 2.5\angle 90° = 6.03\angle 62.04° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.121)

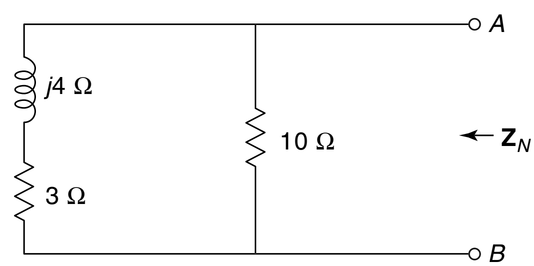

Fig. 6.121

$$\mathbf{Z}_N = \frac{10(3+j4)}{10+3+j4} = 3.68\angle 36.03° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.122)

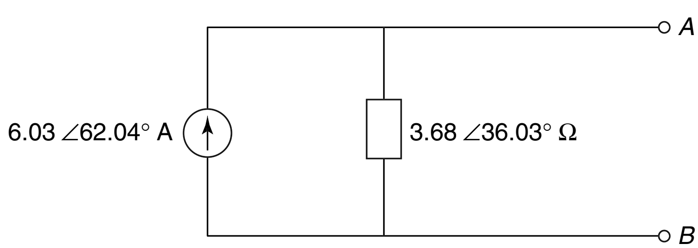

Fig. 6.122

Example 6.43 *Obtain the Norton's equivalent network for Fig. 6.123.*

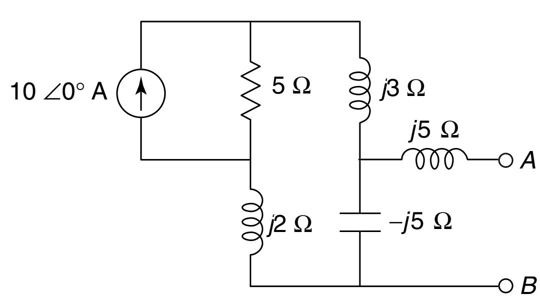

Fig. 6.123

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.124)

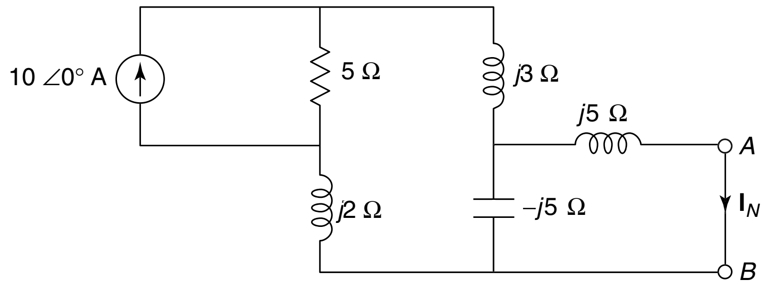

Fig. 6.124

By source transformation, the network can be redrawn as shown in Fig. 6.125.

Writing KVL equations in matrix form,

$$\begin{bmatrix} 5 & j5 \\ j5 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

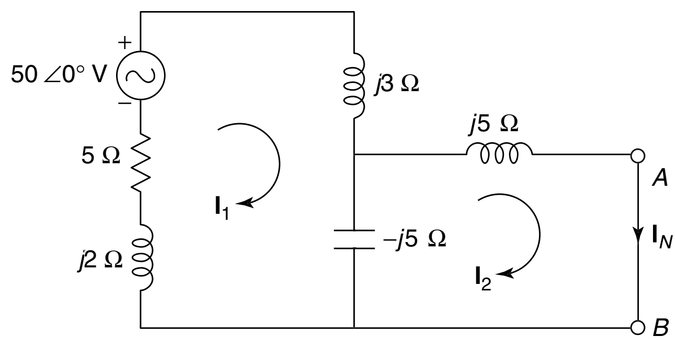

Fig. 6.125

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5 & 50\angle 0^\circ \\ j5 & 0 \end{vmatrix}}{\begin{vmatrix} 5 & j5 \\ j5 & 0 \end{vmatrix}} = 10\angle -90^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 10\angle -90^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.126)

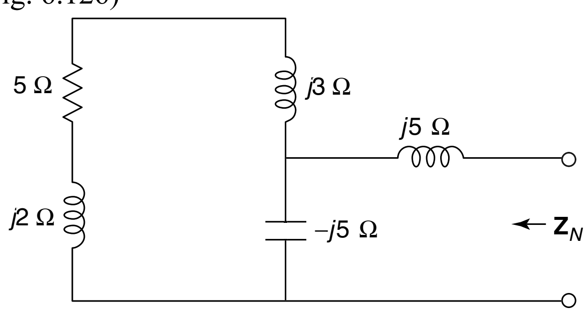

Fig. 6.126

$$\mathbf{Z}_N = j5 + \frac{(5+j5)(-j5)}{5+j5-j5} = 5\ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.127)

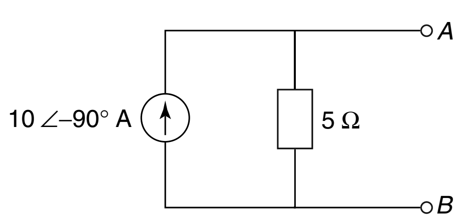

Fig. 6.127

Example 6.44 *Obtain the Norton's equivalent network for Fig. 6.128.*

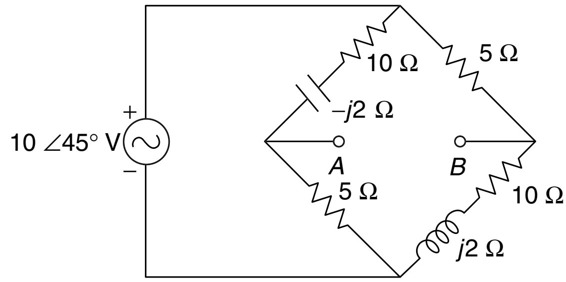

Fig. 6.128

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.129)

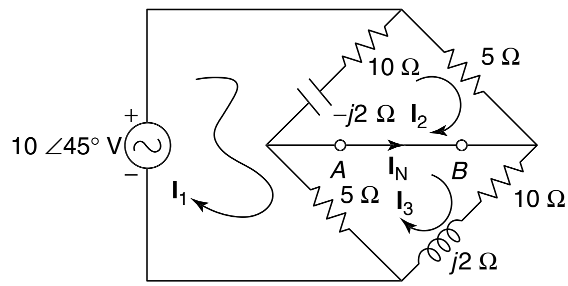

Fig. 6.129

Writing KVL equations in matrix form,

$$\begin{bmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 15-j2 & 10\angle 45^\circ & -5 \\ -10+j2 & 0 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 1\angle 41.28^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 15-j2 & -10+j2 & 10\angle 45^\circ \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 0.49\angle 37.41^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_3 - \mathbf{I}_2 = 0.49\angle 37.41 - 1\angle 41.28^\circ = 0.51\angle -135^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.130)

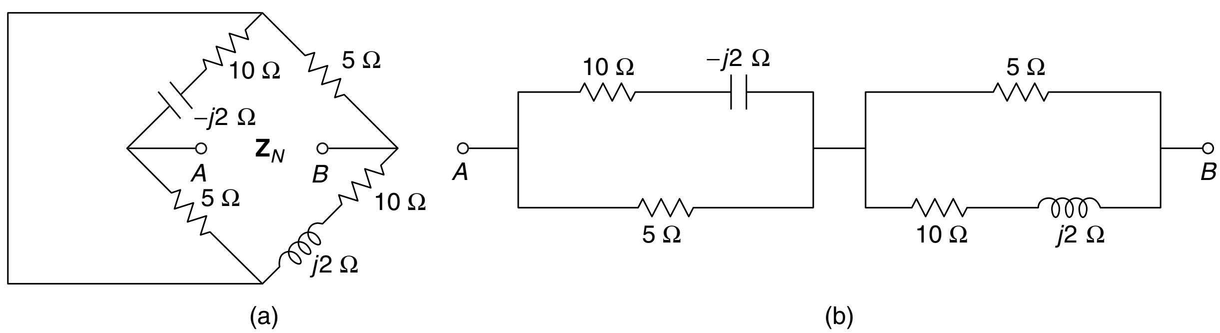

Fig. 6.130

$$\mathbf{Z}_N = \frac{5(10-j2)}{5+10-j2} + \frac{5(10+j2)}{5+10+j2} = 6.72\ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.131)

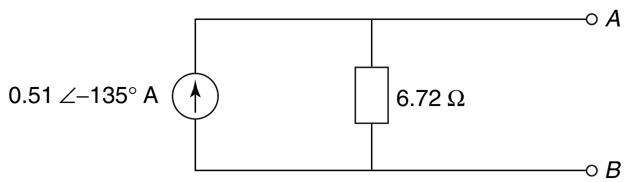

Fig. 6.131

Example 6.45 *Find the current through the 8 Ω resistor in the Network of Fig. 6.132.*

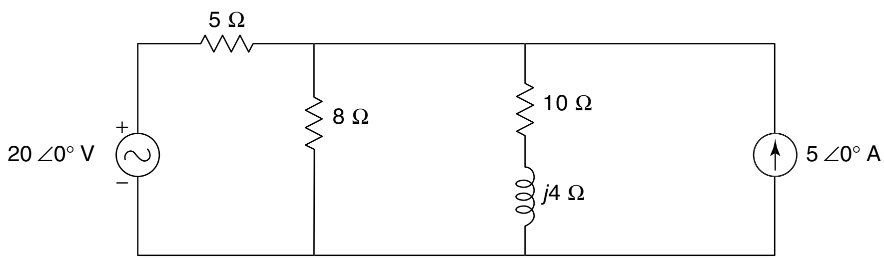

Fig. 6.132

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.133)

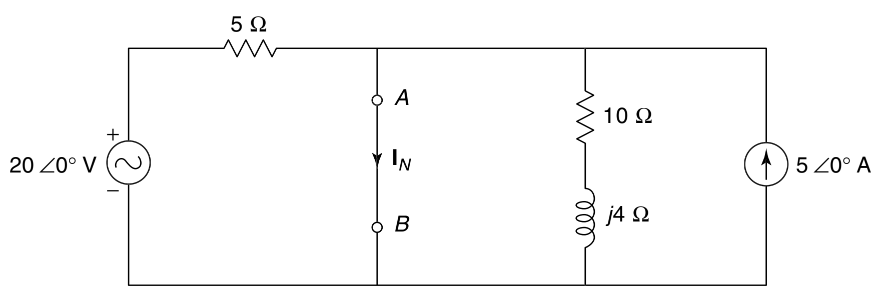

Fig. 6.133

When a short circuit is placed across the $(10+j4)\ \Omega$ impedance, it gets shorted as shown in fig. 6.134.

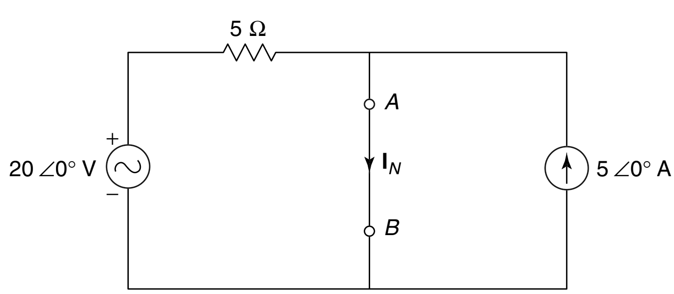

Fig. 6.134

By source transformation, the network is redrawn as shown in Fig. 6.135.

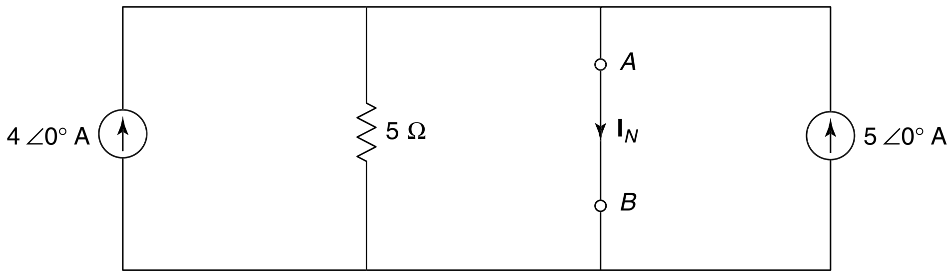

Fig. 6.135

$$\mathbf{I}_N = 4\angle 0° + 5\angle 0° = 9\angle 0° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.136)

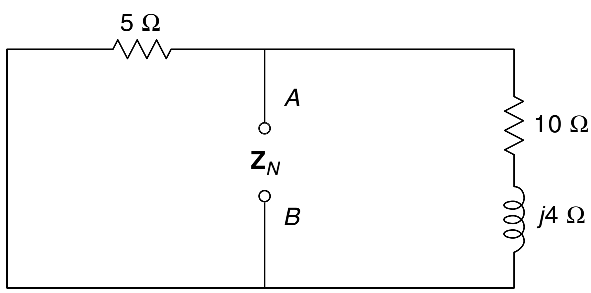

Fig. 6.136

$$\mathbf{Z}_N = \frac{5(10+j4)}{5+10+j4} = 3.47\angle 6.87° \ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.137)

$$\mathbf{I}_L = \frac{9\angle 0°}{3.47\angle 6.87° + 8} = 0.79\angle -2.08° \ \text{A}$$

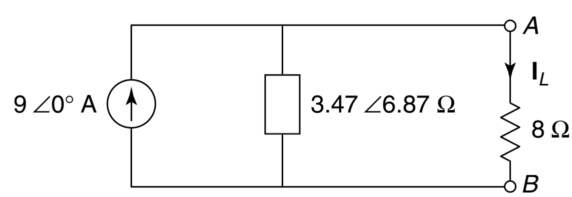

Fig. 6.137

Example 6.46 *Obtain Norton's equivalent network across the terminals A and B in Fig. 6.138.*

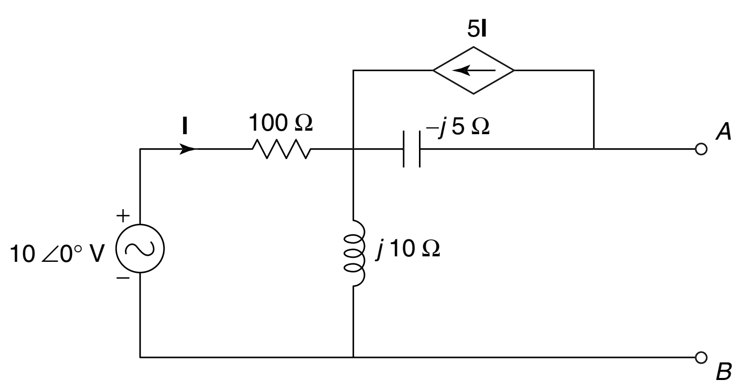

Fig. 6.138

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.139)

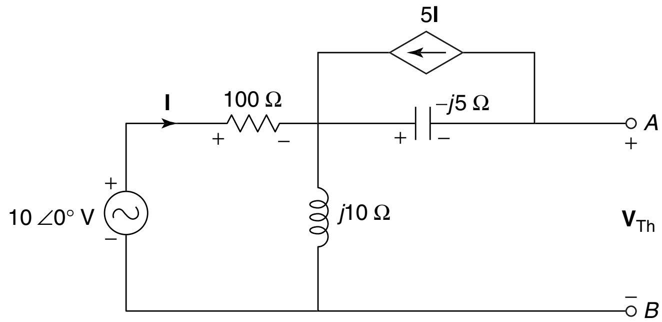

Fig. 6.139

$$\mathbf{I} = \frac{10\angle 0^\circ}{100 + j10} = 0.1\angle -5.71^\circ \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0^\circ - 100\,\mathbf{I} - (-j5)(5\mathbf{I}) - \mathbf{V}_{\text{Th}} = 0$$

$$10\angle 0^\circ - 100(0.1\angle -5.71^\circ) + (j5)(5)(0.1\angle -5.71^\circ) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 3.5\angle 85.1^\circ \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.140)

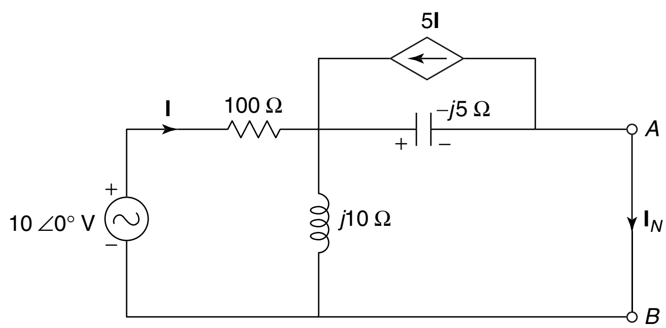

Fig. 6.140

By source transformation, the network is redrawn as shown in Fig. 6.141.

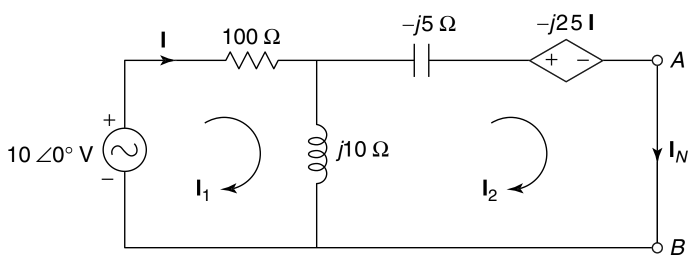

Fig. 6.141

From Fig. 6.141,

$$\mathbf{I} = \mathbf{I}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10\angle 0° - 100\,\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(100 + j10)\,\mathbf{I}_1 - j10\mathbf{I}_2 = 10\angle 0° \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) + j5\mathbf{I}_2 + j25\mathbf{I} = 0$$

$$-j10\mathbf{I}_2 + j10\mathbf{I}_1 + j5\mathbf{I}_2 + j25\mathbf{I}_1 = 0$$

$$j35\mathbf{I}_1 - j5\mathbf{I}_2 = 0 \qquad \text{...(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 100 + j10 & 10\angle 0° \\ j35 & 0 \end{vmatrix}}{\begin{vmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{vmatrix}} = 0.6\angle 30.96° \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 0.6\angle 30.96° \text{ A}$$

Step III Calculation of $\mathbf{Z}_N$

$$\mathbf{Z}_N = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.5\angle 85.1°}{0.6\angle 30.96°} = 5.83\angle 54.14° \ \Omega$$

Step IV Norton's Equivalent Network (Fig. 6.142)

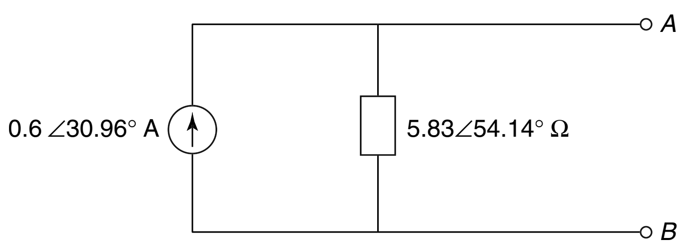

Fig. 6.142

6.7 MAXIMUM POWER TRANSFER THEOREM

This theorem is used to determine the value of load impedance for which the source will transfer maximum power.

Consider a simple network as shown in Fig. 6.143. There are three possible cases for load impedance $\mathbf{Z}_L$.

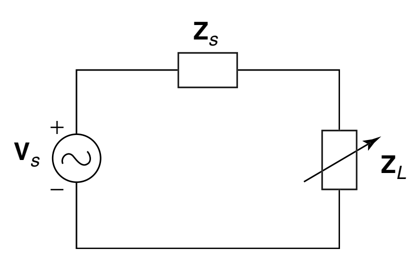

Fig. 6.143

Case (i) When the load impedance is variable resistance (Fig. 6.144)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L} = \frac{\mathbf{V}_s}{R_s + jX_s + R_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

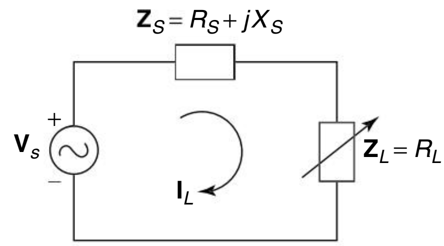

Fig. 6.144 *Purely resistive load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_s|^2 R_L}{\sqrt{(R_s + R_L)^2 + X_s{}^2}}$$

For power to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{\{(R_s + R_L)^2 + X_s^2\} - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + X_s^2]^2}\right] = 0$$

$$(R_s + R_L)^2 + X_s^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + X_s^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + X_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + X_s^2$$

$$R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$$

Hence, load resistance R_L should be equal to the magnitude of the source impedance for maximum power transfer.

Case (ii) When the load impedance is a complex impedance with variable resistance and variable reactance (Fig. 6.145)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

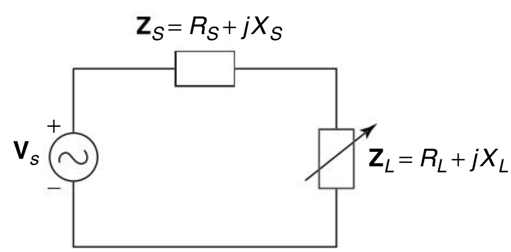

Fig. 6.145 *Complex impedance load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

For maximum value of P_L, denominator of the equation should be small, ie. $X_L = -X_s$.

$$P_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2}$$

Differentiating the above equation w.r.t. R_L and equating to zero,

$$\frac{dP_L}{dR_L} = |\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 - 2\,R_L\,(R_s + R_L)}{(R_s + R_L)^2} \right] = 0$$

$$(R_s + R_L)^2 - 2\,R_L\,(R_s + R_L) = 0$$

$$R_s^2 + 2\,R_s R_L + R_L^2 - 2\,R_L R_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

Hence, load resistance R_L should be equal to source resistance R_L and load reactance X_L should be equal to negative value of source reactance for maximum power transfer.

$$\mathbf{Z}_L = \mathbf{Z}_s^* = R_s - jX_s$$

i.e. load impedance should be a complex conjugate of the source impedance.

Case (iii) When the load impedance is a complex impedance with variable resistance and fixed reactance (Fig. 6.146)

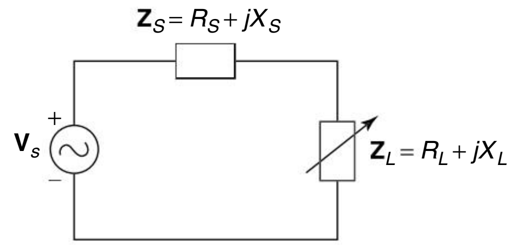

Fig. 6.146 *Complex impedance load*

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2\, R_L = \frac{|\mathbf{V}_s|^2\, R_L}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 + (X_s + X_L)^2 - 2\,R_L(R_s + R_L)}{\{(R_s + R_L)^2 + (X_s + X_L)^2\}^2} \right] = 0$$

$$(R_s + R_L)^2 + (X_s + X_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^2 + 2\,R_s\,R_L + R_L^2 + (X_s + X_L)^2 - 2\,R_L R_s - 2\,R_L^2 = 0$$

$$R_s^2 + (X_s + X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s + X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$= |R_s + j(X_s + X_L)|$$

$$= |R_s + jX_s + jX_L|$$

$$= |Z_s + jX_L|$$

Hence, load resistance R_L should be equal to the magnitude of the impedance $\mathbf{Z}_s + jX_L$, i.e. $|\mathbf{Z}_s + jX_L|$ for maximum power transfer.

Example 6.47 *For maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 6.147 if (i) $\mathbf{Z}_L$ is an impedance, and (ii) $\mathbf{Z}_L$ is pure resistance.*

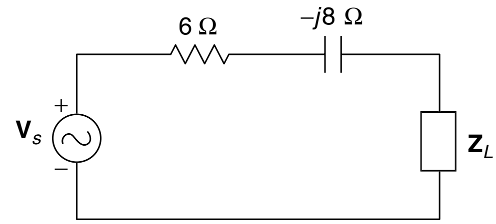

Fig. 6.147

Solution

$$\mathbf{Z}_s = (6 - j8)\ \Omega$$

(i) If $\mathbf{Z}_L$ is an impedance

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_s^* = (6 + j8)\ \Omega$

(ii) If $\mathbf{Z}_L$ is a resistance

For maximum power transfer, $\mathbf{Z}_L = |\mathbf{Z}_S| = |6 + j8| = 10\ \Omega$

Example 6.48 *For the maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 6.148 for the following cases:*

(i) $\mathbf{Z}_L$ is variable resistance, (ii) $\mathbf{Z}_L$ is complex impedance, with variable resistance and variable reactance, and (iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of j5 Ω.

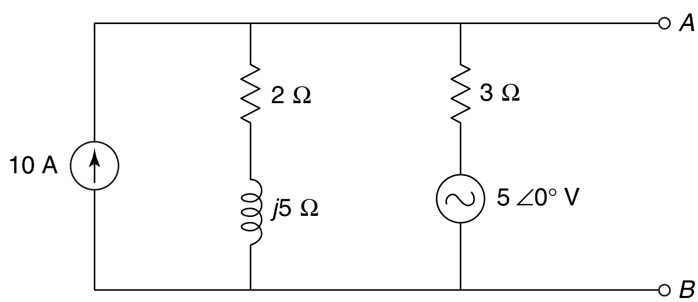

Fig. 6.148

Solution Thevenin's impedance can be calculated by replacing voltage source by a short circuit and current source by an open circuit.

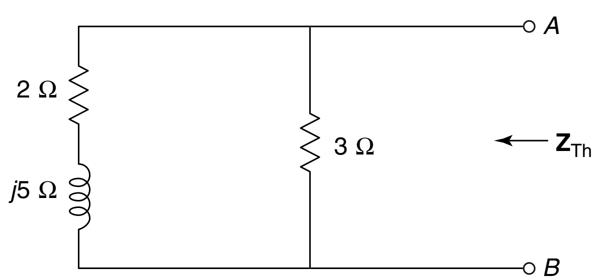

Fig. 6.149

$$\mathbf{Z}_{\text{Th}} = \frac{3(2+j5)}{3+2+j5} = (2.1+j0.9)\ \Omega$$

For maximum power transfer, value of $\mathbf{Z}_L$ will be,

(i) $\mathbf{Z}_L$ is variable resistance

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}}| = |2.1+j0.9| = 2.28\ \Omega$$

(ii) $\mathbf{Z}_L$ is complex impedance with variable resistance and variable reactance

$$\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = (2.1-j0.9)\ \Omega$$

(iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of $j5\ \Omega$

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}} + j5| = |2.1+j\,0.9+j5| = 6.26\ \Omega$$

Example 6.49 *Find the impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 6.150. Find maximum power.*

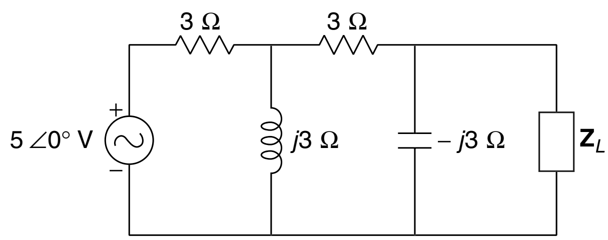

Fig. 6.150

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.151)

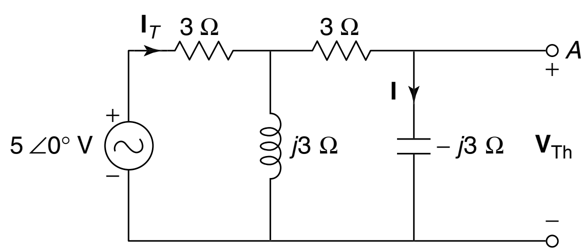

Fig. 6.151

$$\mathbf{Z}_T = 3 + \frac{j3(3-j3)}{3+j3-j3} = 6.71\angle 26.57°\ \Omega$$

$$\mathbf{I}_T = \frac{5\angle 0°}{6.71\angle 26.57°} = 0.75\angle -26.57°\ \text{A}$$

By current division rule,

$$\mathbf{I} = 0.75\angle -26.57° \times \frac{j3}{3+j3-j3} = 0.75\angle 63.43°\ \text{A}$$

$$\mathbf{V}_{\text{Th}} = (-j3)(0.75\angle 63.43°) = 2.24\angle -26.57°\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.152)

$$\mathbf{Z}_{Th} = [(3 \parallel j3) + 3] \parallel (-j3)$$
$$= 3\ \angle -53.12°\ \Omega$$
$$= (1.8 - j2.4)\ \Omega$$

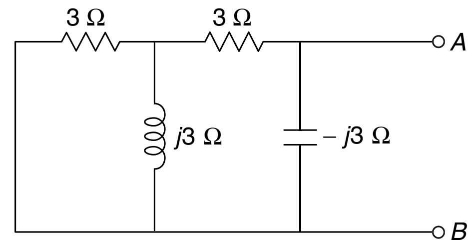

Fig. 6.152

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be a complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.8 + j2.4)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.153)

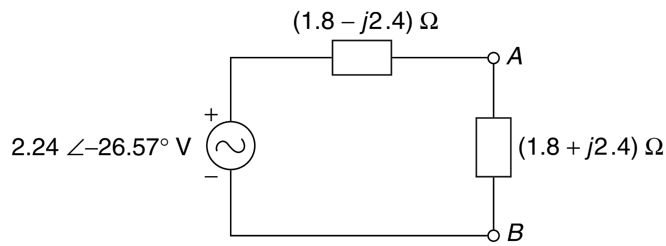

Fig. 6.153

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|2.24|^2}{4 \times 1.8} = 0.7\ \text{W}$$

Example 6.50 *Find the value of* $\mathbf{Z}_L$ *for maximum power transfer in the network shown and find maximum power.*

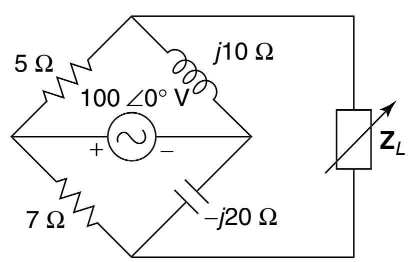

Fig. 6.154

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.155)

$$\mathbf{I}_1 = \frac{100\angle 0°}{5 + j10} = 8.94\angle -63.43°\ \text{A}$$

$$\mathbf{I}_2 = \frac{100\angle 0°}{7 - j20} = 4.72\angle 70.7°\ \text{A}$$

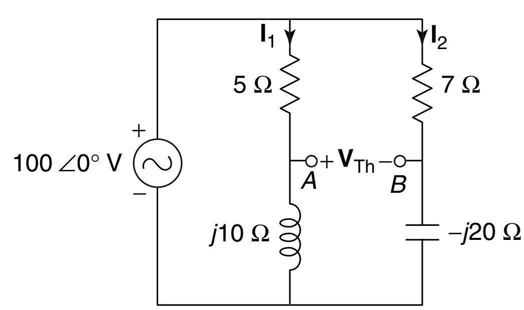

Fig. 6.155

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_A - \mathbf{V}_B = (8.94\angle -63.43°)(j10) - (4.72\angle 70.7°)(-j20) = 71.76\angle 97.3° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.156)

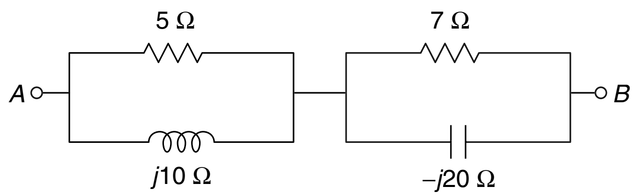

Fig. 6.156

$$\mathbf{Z}_{\text{Th}} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{7-j20} = \frac{50\angle 90°}{11.18\angle 63.43°} + \frac{140\angle -90°}{21.19\angle -70.7°} = (10.23 - j0.18)\ \Omega$$

Step III For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (10.23 + j0.18)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.157)

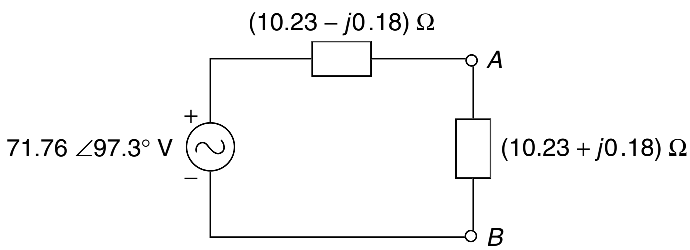

Fig. 6.157

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|71.76|^2}{4 \times 10.23} = 125.84 \text{ W}$$

Example 6.51 *Find the value of load impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 6.158. Find maximum power.*

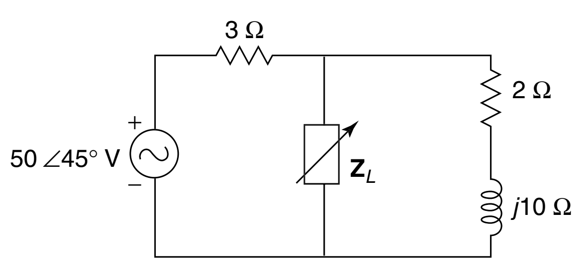

Fig. 6.158

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.159)

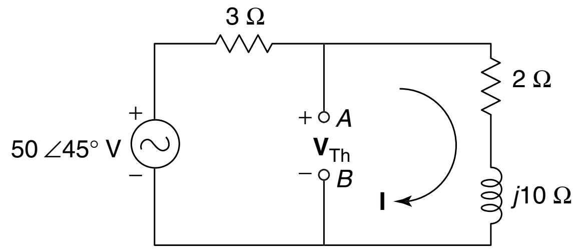

Fig. 6.159

$$\mathbf{I} = \frac{50\angle 45^\circ}{3+2+j10} = 4.47\angle -18.43^\circ \text{ A}$$

$$\mathbf{V}_{Th} = (2+j10)\,\mathbf{I} = (2+j10)(4.47\angle -18.43^\circ) = 45.6\angle 60.26^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.160)

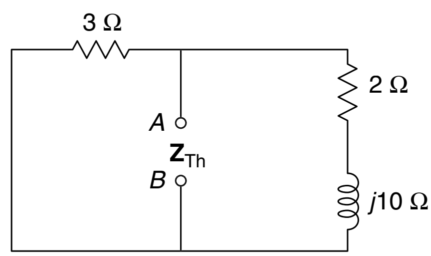

Fig. 6.160

$$\mathbf{Z}_{Th} = \frac{3(2+j10)}{3+2+j10} = (2.64+j0.72)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (2.64 - j0.72)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.161)

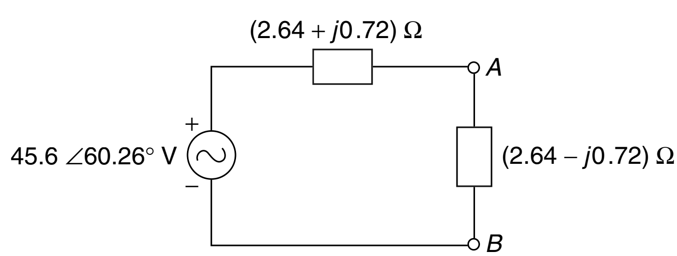

Fig. 6.161

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|45.6|^2}{4\times 2.64} = 196.91 \text{ W}$$

Example 6.52 *Determine the load $\mathbf{Z}_L$ required to be connected in the network of Fig. 6.162 for maximum power transfer. Determine the maximum power drawn.*

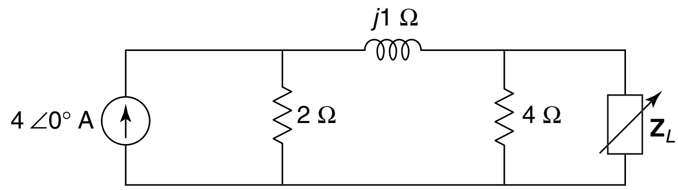

Fig. 6.162

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.163)

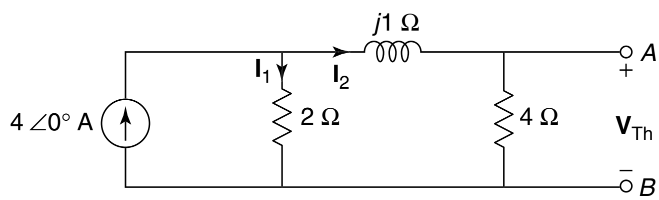

Fig. 6.163

$$\mathbf{I}_2 = 4\angle 0° \times \frac{2}{6+j1} = 1.315\angle -9.46° \text{ A}$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 = 4(1.315\angle -9.46°) = 5.26\angle -9.46° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.164)

$$\mathbf{Z}_{\text{Th}} = \frac{4(2+j1)}{4+2+j1} = 1.47\angle 17.1° = (1.41 + j0.43)\ \Omega$$

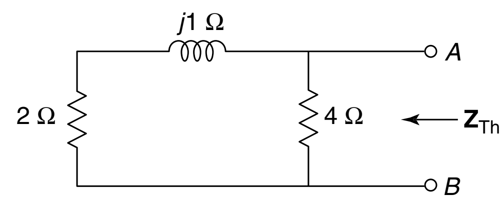

Fig. 6.164

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.41 - j0.43)\ \Omega$$

Step IV Calculation of $P_{\max}$ (Fig. 6.165)

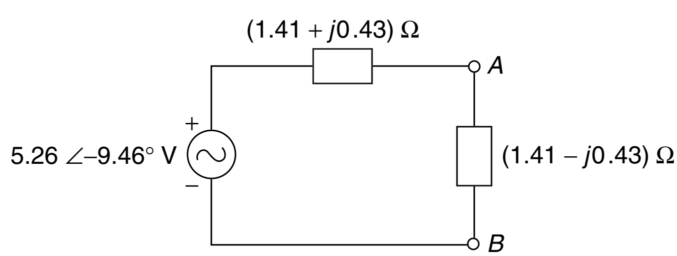

Fig. 6.165

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|5.26|^2}{4 \times 1.41} = 4.91 \text{ W}$$

Example 6.53 *In the network shown in Fig. 6.166, find the value of $\mathbf{Z}_L$ for which the power transferred will be maximum. Also find maximum power.*

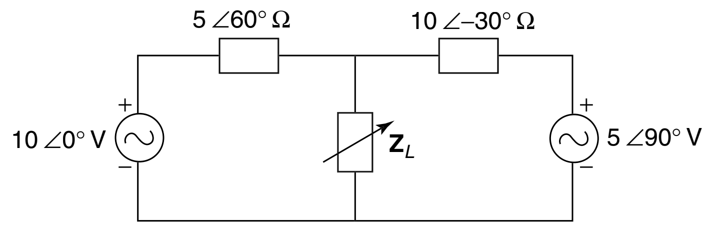

Fig. 6.166

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.167)

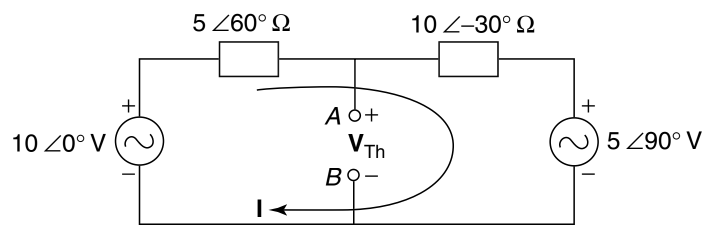

Fig. 6.167

Applying KVL to the mesh,

$$10\angle 0° - (5\angle 60°)\mathbf{I} - (10\angle -30°)\mathbf{I} - 5\angle 90° = 0$$
$$11.18\angle -26.57° - (11.18\angle -3.43°)\mathbf{I} = 0$$
$$\mathbf{I} = 1\angle -23.14° \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0° - (5\angle 60°)\mathbf{I} - \mathbf{V}_{\text{Th}} = 0$$
$$10\angle 0° - (5\angle 60°)(1\angle -23.14°) - \mathbf{V}_{\text{Th}} = 0$$
$$\mathbf{V}_{\text{Th}} = 6.71\angle -26.56° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.168)

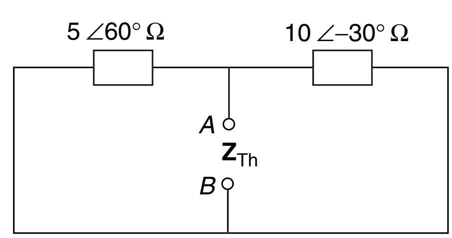

Fig. 6.168

$$\mathbf{Z}_{\text{Th}} = \frac{(5\angle 60°)(10\angle -30°)}{5\angle 60° + 10\angle -30°} = 4.47\angle 33.43° \ \Omega = (3.73 + j2.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$Z_L = Z_{Th}^* = (3.73 - j2.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.169)

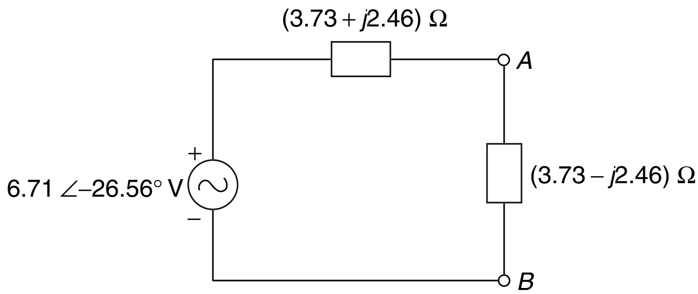

Fig. 6.169

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{(6.71)^2}{4 \times 3.73} = 3.02\ \text{W}$$

Example 6.54 *In the network shown in Fig. 6.170, find the value of $\mathbf{Z}_L$ so that power transfer from the source is maximum. Also find maximum power.*

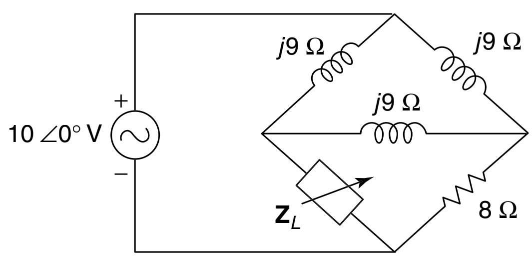

Fig. 6.170

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.171)

Applying Star-delta transformation (Fig. 6.172)

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \frac{(j9)(j9)}{j9 + j9 + j9} = j3\ \Omega$$

$\mathbf{V}_{Th}$ = Voltage drop across $(8 + j3)\Omega$ impedence

$$= (8 + j3)\frac{10\angle 0^\circ}{8 + j3 + j3} = 8.54\angle -16.31^\circ\ \text{V}$$

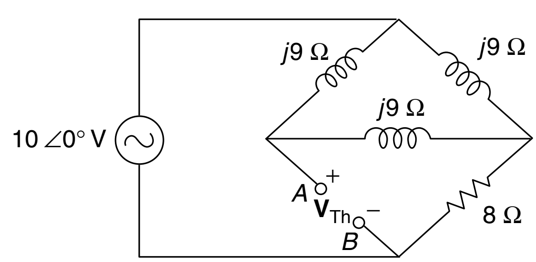

Fig. 6.171

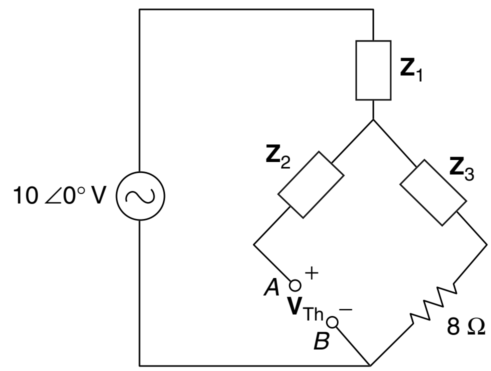

Fig. 6.172

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.173)

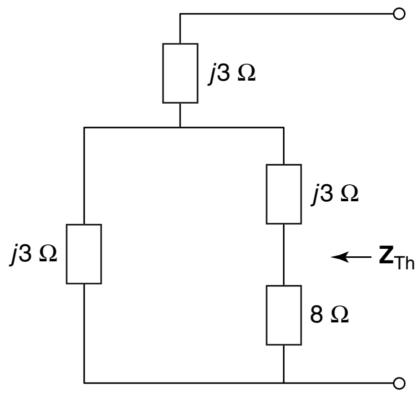

Fig. 6.173

$$\mathbf{Z}_{\text{Th}} = j3 + \frac{j3(8+j3)}{j3+8+j3)} = 5.51\angle 82.49°\ \Omega = (0.72 + j5.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = (0.72 - j5.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.174)

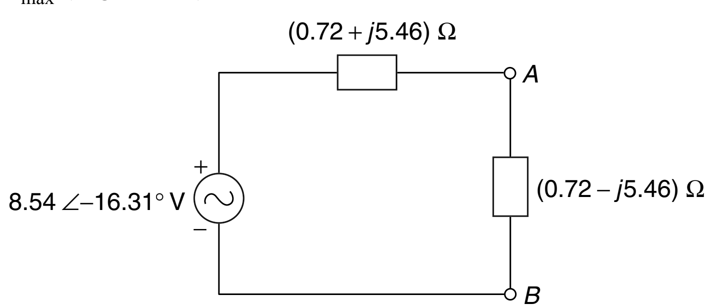

Fig. 6.174

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{(8.54)^2}{4\times 0.72} = 25.32\ \text{W}$$

Example 6.55 *For the network shown in Fig. 6.175, find the value of $\mathbf{Z}_L$ that will transfer maximum power from the source. Also find maximum power.*

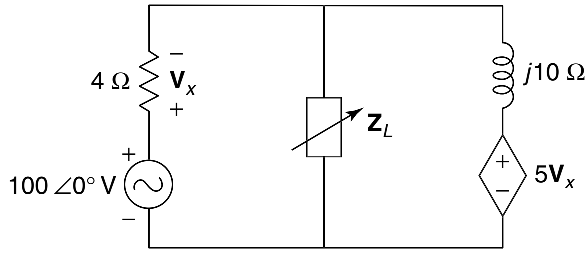

Fig. 6.175

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.176)

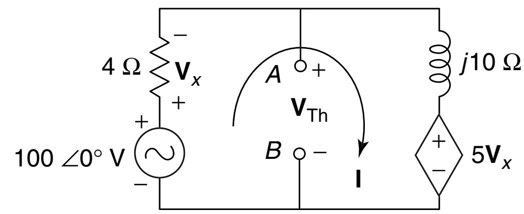

Fig. 6.176

From Fig 6.176,

$$\mathbf{V}_x = 4\mathbf{I}$$

Applying KVL to the mesh,

$$100\angle 0° - 4\mathbf{I} - j\,10\,\mathbf{I} - 5\mathbf{V}_x = 0$$

$$100\angle 0° - (4 + j10)\,\mathbf{I} - 5(4\mathbf{I}) = 0$$

$$\mathbf{I} = \frac{100\ \angle 0°}{24 + j10} = 3.85\ \angle -22.62°\ \text{A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$100\ \angle\ 0° - 4\mathbf{I} - \mathbf{V}_{\text{Th}} = 0$$

$$100\angle 0° - 4(3.85\ \angle -22.62°) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 86\angle 3.95°\ \text{V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.177)

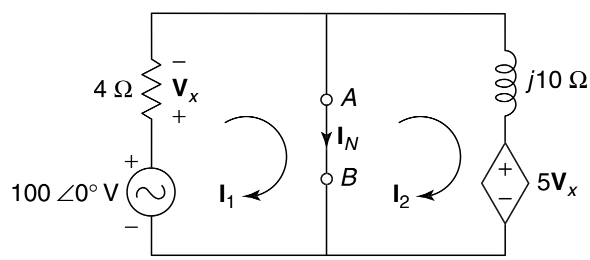

Fig. 6.177

From Fig. 6.177,

$$\mathbf{V}_x = 4\mathbf{I}_1$$

Applying KVL to Mesh 1,

$$100\ \angle 0° - 4\mathbf{I}_1 = 0$$

$$\mathbf{I}_1 = 25\ \text{A}$$

Applying KVL to Mesh 2,

$$-j10\mathbf{I}_2 - 5\mathbf{V}_x = 0$$

$$-j10\mathbf{I}_2 - 5(4\mathbf{I}_1) = 0$$

$$-j10\mathbf{I}_2 - 5(100) = 0$$

$$\mathbf{I}_2 = 50\angle 90°\text{A}$$

$$\mathbf{I}_N = \mathbf{I}_1 - \mathbf{I}_2 = 25 - 50\angle 90° = 55.9\angle -63.43°\text{A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{86\angle 3.95°}{55.9\angle -63.43°} = 1.54\angle 67.38°\ \Omega = (0.59 + j1.42)\ \Omega$$

Step IV Calculation of $\mathbf{Z}_L$

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{\text{Th}}^* = (0.59 - j1.42)\ \Omega$

Step V Calculation of $P_{\max}$ (Fig. 6.178)

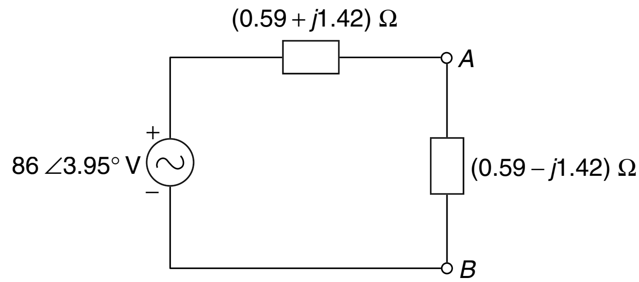

Fig. 6.178

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{(86)^2}{4 \times 0.59} = 3133.9 \text{ W}$$

6.8 RECIPROCITY THEOREM

The Reciprocity theorem states that '*In a linear, bilateral, active, single-source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.*'

Example 6.56 *Find the current through the 6 Ω resistor and verify the reciprocity theorem.*

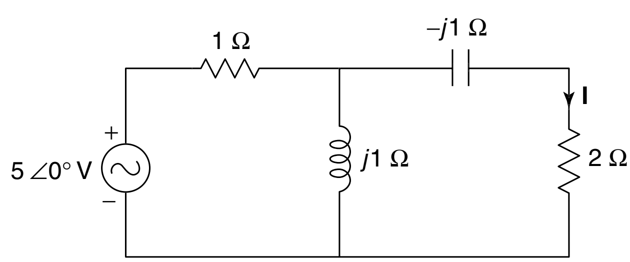

Fig. 6.179

Solution

Case I Calculation of current **I** when excitation and response are not interchanged (Fig. 6.180)

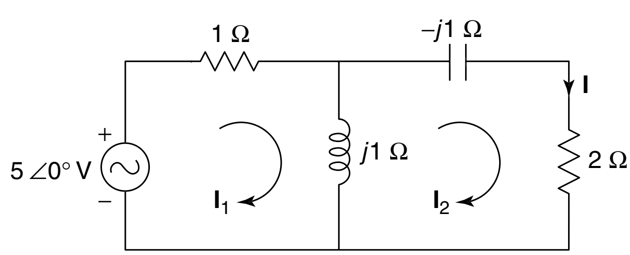

Fig. 6.180

Applying KVL to Mesh 1,

$$5\angle 0° - 1\mathbf{I}_1 - j1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(1 + j1)\,\mathbf{I}_1 - j1\mathbf{I}_2 = 5\angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2-\mathbf{I}_1)+j1\mathbf{I}_2-2\mathbf{I}_2=0$$

$$-j1\mathbf{I}_1+2\mathbf{I}_2=0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}1+j1 & -j1\\ -j1 & 2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}5\angle 0^\circ\\ 0\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2=\frac{\begin{vmatrix}1+j1 & 5\angle 0^\circ\\ -j1 & 0\end{vmatrix}}{\begin{vmatrix}1+j1 & -j1\\ -j1 & 2\end{vmatrix}}=1.39\angle 56.31^\circ\text{ A}$$

$$\mathbf{I}=\mathbf{I}_2=1.39\angle 56.31^\circ\text{ A}$$

Case II Calculation of current **I** when excitation and response are interchanged (Fig. 6.181)

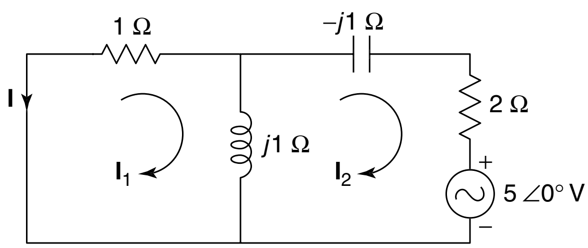

Fig. 6.181

Applying KVL to Mesh 1,

$$-1\mathbf{I}_1-j1(\mathbf{I}_1-\mathbf{I}_2)=0$$

$$(1+j1)\mathbf{I}_1-j1\mathbf{I}_2=0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2-\mathbf{I}_1)+j1\mathbf{I}_2-2\mathbf{I}_2-5\angle 0^\circ=0$$

$$-j1\mathbf{I}_1+2\mathbf{I}_2=-5\angle 0^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}1+j1 & -j1\\ -j1 & 2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}0\\ -5\angle 0^\circ\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1=\frac{\begin{vmatrix}0 & -j1\\ -5\angle 0^\circ & 2\end{vmatrix}}{\begin{vmatrix}1+j1 & -j1\\ -j1 & 2\end{vmatrix}}=1.38\angle -123.69^\circ\text{ A}$$

$$\mathbf{I}=-\mathbf{I}_1=1.39\angle 56.31^\circ\text{ A}$$

Since the current **I** is same in both the cases, the reciprocity theorem is verified.

Example 6.57 *In the network of Fig. 6.182, find the voltage $\mathbf{V}_x$ and verify the reciprocity theorem.*

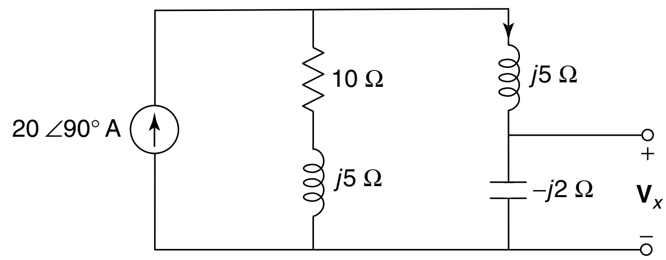

Fig. 6.182

Solution

Case I Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged. (Fig. 6.183)

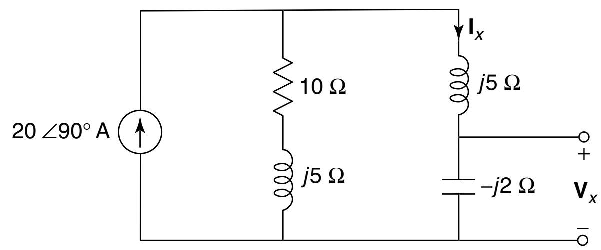

Fig. 6.183

By current division rule,

$$\mathbf{I}_x = (20\angle 90°)\frac{(10+j5)}{(10+j5)+(j5-j2)} = 17.46\angle 77.91° \text{ A}$$

$$\mathbf{V}_x = (-j2)\mathbf{I}_x = (-j2)(17.46\angle 77.91°) = 34.92\ \angle -12.09° \text{ V}$$

Case II Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged (Fig. 6.184)

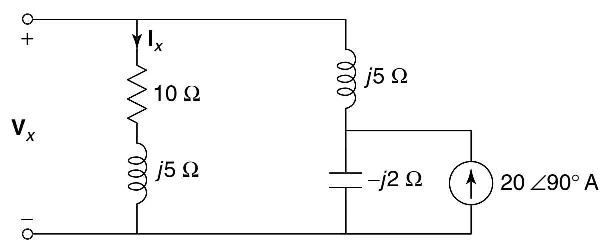

Fig. 6.184

$$\mathbf{I}_x = (20\angle 90°)\frac{(-j2)}{(-j2)+(10+j5+j5)} = 3.12\angle -38.66° \text{A}$$

$$\mathbf{V}_x = (10+j5)\mathbf{I}_x = (10+j5)(3.12\angle -38.66°) = 34.88\angle -12.09° \text{V}$$

Since the voltage $\mathbf{V}_x$ is same in both the cases, the reciprocity theorem is verified.